Stage 4 ★
Mixed Selection 1 – Solutions

1. **Below 400**
   Note that the number at the end of the nth row is n^2, so 400 will lie at the end of the 20th row. The row below will end in 21^2, i.e. 441, so the number directly below 400 will be 440.

2. **Big Fibonacci**
   Let the first two terms of the sequence be a and b respectively.

   Then the next three terms are $a+b$, $a+2b$, $2a+3b$. So $2a+3b=2004$.

   For a to be as large as possible, we need b to be as small as possible, consistent with both being positive integers.

   If $b=1$ then $2a=2001$, but a is an integer, so $b\neq 1$.

   However, if $b=2$ then $2a=1998$, so the maximum possible value of a is 999.

3. **What a coincidence!**
   The sequences have common differences of 7 and 9 respectively. The lowest common multiple of 7 and 9 is 63, so the next term after 2005 to appear in both sequences is 2005 + 63, that is 2068.

4. **Alternating sum**
   We can write the series as:
   $1+(-2+3)+(-4+5)+\ldots+(-(n-1)+n)$. So the sum is the same as $1+1+\ldots+1=\frac{n+1}{2}$.
   So $(n+1)=4016$, so $n=4015$.

5. **Collatz-ish**
   The terms of the sequence are:
   6,3,14,7,34,17,84,42,21,104,52,26,13,64,32,16,8,4,2,1,4,2,1,…
   As can be seen, there will now be no other terms in the sequence other than 4, 2, and 1. It can also be seen that the only values of n for which the nth term equals n are 13 and 16.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)