



# Creating and Manipulating Expressions

## Stage 3 ★

### Mixed Selection 1 – Solutions

#### 1. Inspector Remorse

Solving two murders takes  $2x$  hours, six car thefts takes  $6 \times \frac{x}{6} = x$ ,  
and 4 bank robberies takes  $4 \times \frac{x}{2} = 2x$ .

Total =  $2x + 2x + x = 5x$  hours.

#### 2. Multiple magic

Call your first number  $x$ . Doubling and adding five gives  $2x + 5$ .

Doubling and adding two gives  $2(2x + 5) + 2 = 4x + 12$ .

Subtracting  $x$  then leaves  $3x + 12 = 3(x + 4)$  which is always a multiple of 3.

#### 3. Square and cube

Let the number be  $x$ . Then  $x^2 = 2x^3$ , that is  $x^2(2x - 1) = 0$ . So

$x = 0$  or  $x = \frac{1}{2}$ . However,  $x$  is positive, so the only solution is  $x = \frac{1}{2}$ .

#### 4. Forming groups

Let the number of Boys in group 1 be  $x$ . The number of girls in group 1 is therefore  $40 - x$ . and the number of girls in group 2 is

$$33 - (40 - x) = x - 7.$$

There are therefore 7 fewer girls in group 2 than boys in group 1.

#### 5. No matter

Let the number thought of be  $x$ . Then the final number is

$$4(2x + 3) - 5 - x = 7x + 7 = 7(x + 1).$$

#### 6. Nine in a line

The average of the 9 numbers is 223, so they are

219, 220, ..., 226, 227. So the largest number is 227.

It may be tempting to use algebra here. Calling the numbers  $x, x + 1, \dots, x + 8$ . Then summing the numbers to get  $9x + 36$ . Setting this equal to 2007 and solving for  $x$  gives  $x = 219$ , giving the last number  $x + 8 = 227$ . It is slightly easier to consider the numbers  $n - 4, n - 3, n - 2, n - 1, n, n + 1, n + 2, n + 3, n + 4$ . Then, summing them, we get  $9n = 2007$  which gives us  $n = 223$  and so the largest number (now  $n + 4$ ) is 227.

*These problems are adapted from UKMT Mathematical Challenge problems ([ukmt.org.uk](http://ukmt.org.uk)).*