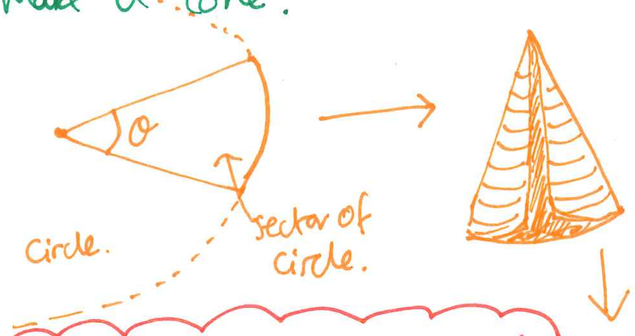
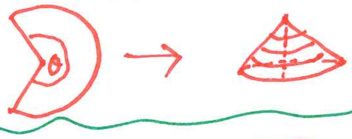
HATS

Wizard's hat

You need a sector of a circle. Stick together the two straight edges to make a cone.



If you make the angle θ of the sector larger you will get a shorter, wider hat.

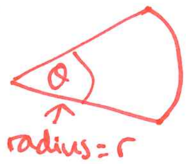


Witch's hat

Make a wizards hat and then stick a ring on the bottom:

How large does the ring need to be?

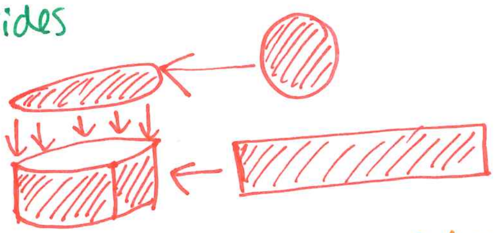
The circumference of the **hole** in the ring needs to be the same length as the bottom of the wizards hat. This is the arc-length of the sector.



arc length $= \frac{\theta}{360} \times 2\pi r$.

Fez

You need a circle for the top and a rectangle which you can roll up into an open cylinder for the sides

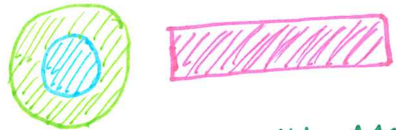


How long does the rectangle need to be?

The width of the rectangle needs to be the circumference of the circle and the height of the rectangle should be the height you want your hat to be.

Top hat

This is a fez with a ring on the bottom. The hole in the ring is the same size as the circle on the top. So we can cut out the pieces like this:



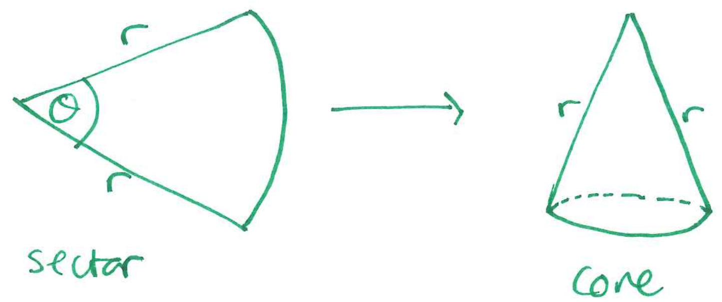
and these will make a hat like this:

These colours make a much prettier hat than the black one!

What is the tallest witch's hat you can make from a sheet of A3 paper?

Firstly I want to be able to find the height of a cone.

Suppose the sector has angle θ and is from a circle with radius r



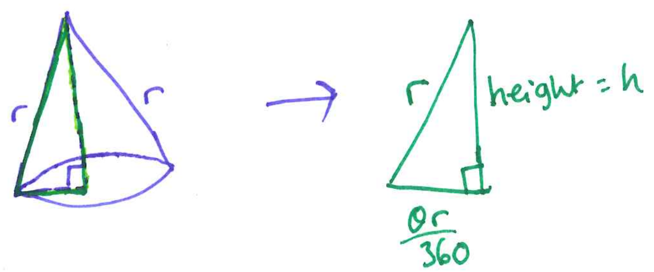
The arc length of the sector is $\boxed{\frac{\theta}{360} \times 2\pi r}$

This is the circumference of the circle at the base of the cone.

Therefore the radius of the base of the cone is

$$\frac{1}{2\pi}\left(\frac{\theta}{360} \times 2\pi r\right) = \frac{\theta r}{360}.$$

Take a cross section of the cone:



Using Pythagoras' Theorem we can find the height (h):

$$r^2 = h^2 + \left(\frac{\theta r}{360}\right)^2$$

$$h^2 = r^2 - \left(\frac{\theta r}{360}\right)^2$$

$$h^2 = r^2\left(1 - \left(\frac{\theta}{360}\right)^2\right)$$

$$h = r\sqrt{1 - \left(\frac{\theta}{360}\right)^2}$$

(take the positive square root because h is a length).

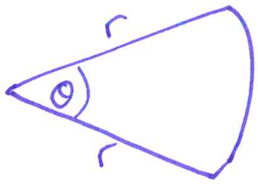
So the height of a cone made from a sector of a circle with angle θ and radius r is

$$\boxed{r\sqrt{1-\left(\frac{\theta}{360}\right)^2}}$$

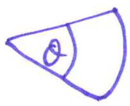
- If we increase r then the height will increase.
- If we decrease θ then $\left(\frac{\theta}{360}\right)^2$ will decrease, so $\sqrt{1-\left(\frac{\theta}{360}\right)^2}$ will increase, and so height will increase.

So to make the tallest cone we need to make r as <u>big</u> as possible and θ as <u>small</u> as possible.

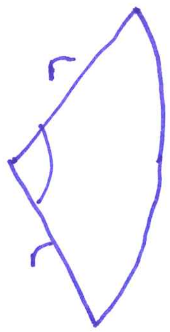
For example,



would make a taller hat than

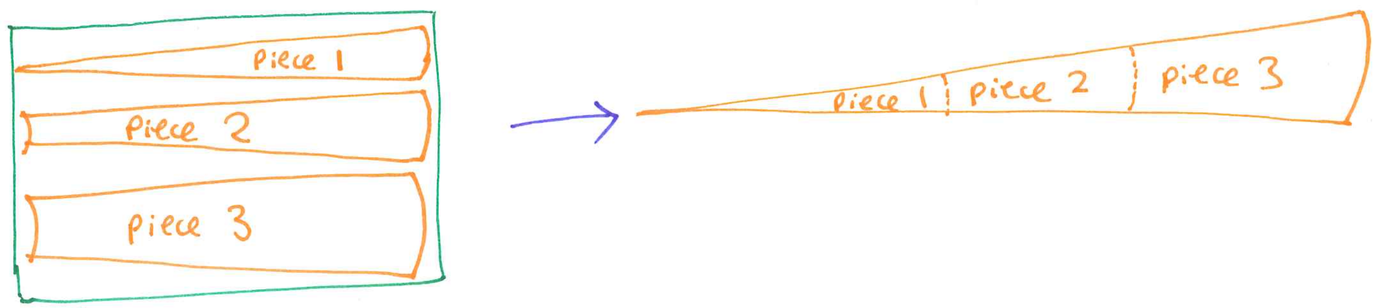


or



I think you can make a cone of <u>any</u> height from an A3 sheet of paper because you could cut out lots of thin strips of paper and stick them together to make a really long thin sector

e.g.



A3 paper

Therefore I think you could make a cone of height 1 metre, or 10 metres or 100000 km high from a single sheet of A3 – but I think it would be hard to do in real life!