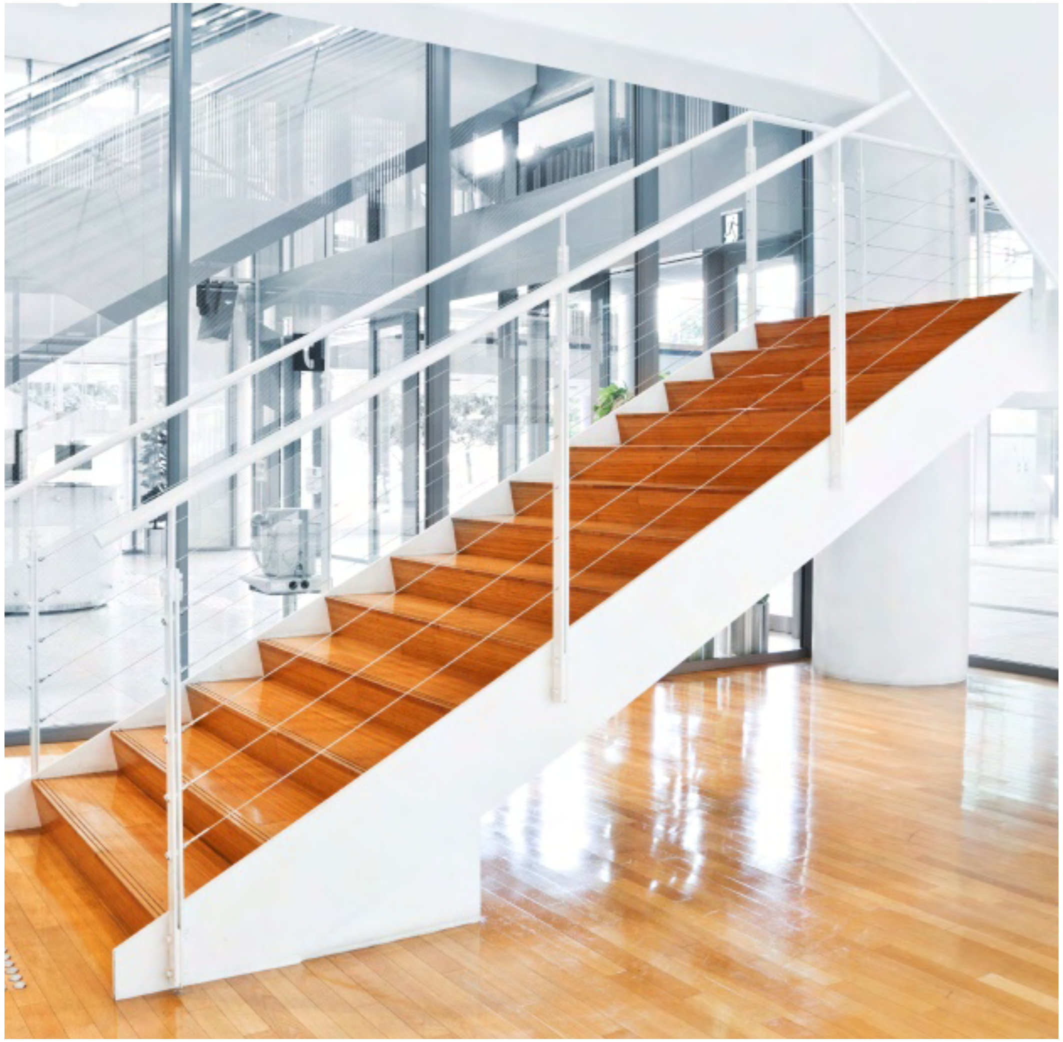
1 Step, 2 Step



Liam's house has a staircase with 12 steps. He can go down the steps one at a time or two at a time.

For example, he could go down 1 step, then 1 step, then 2 steps, then 2, 2, 1, 1, 1, 1.

In how many different ways can Liam go down the 12 steps, taking 1 or 2 steps at a time?

5 on the Clock

On a digital clock showing 24-hour time, over a whole day, how many times does a 5 appear?

Is it the same number for a 12-hour clock over a whole day?

American Billions

Find the ten-digit number which uses each of the digits 0 to 9 once and has the following properties:

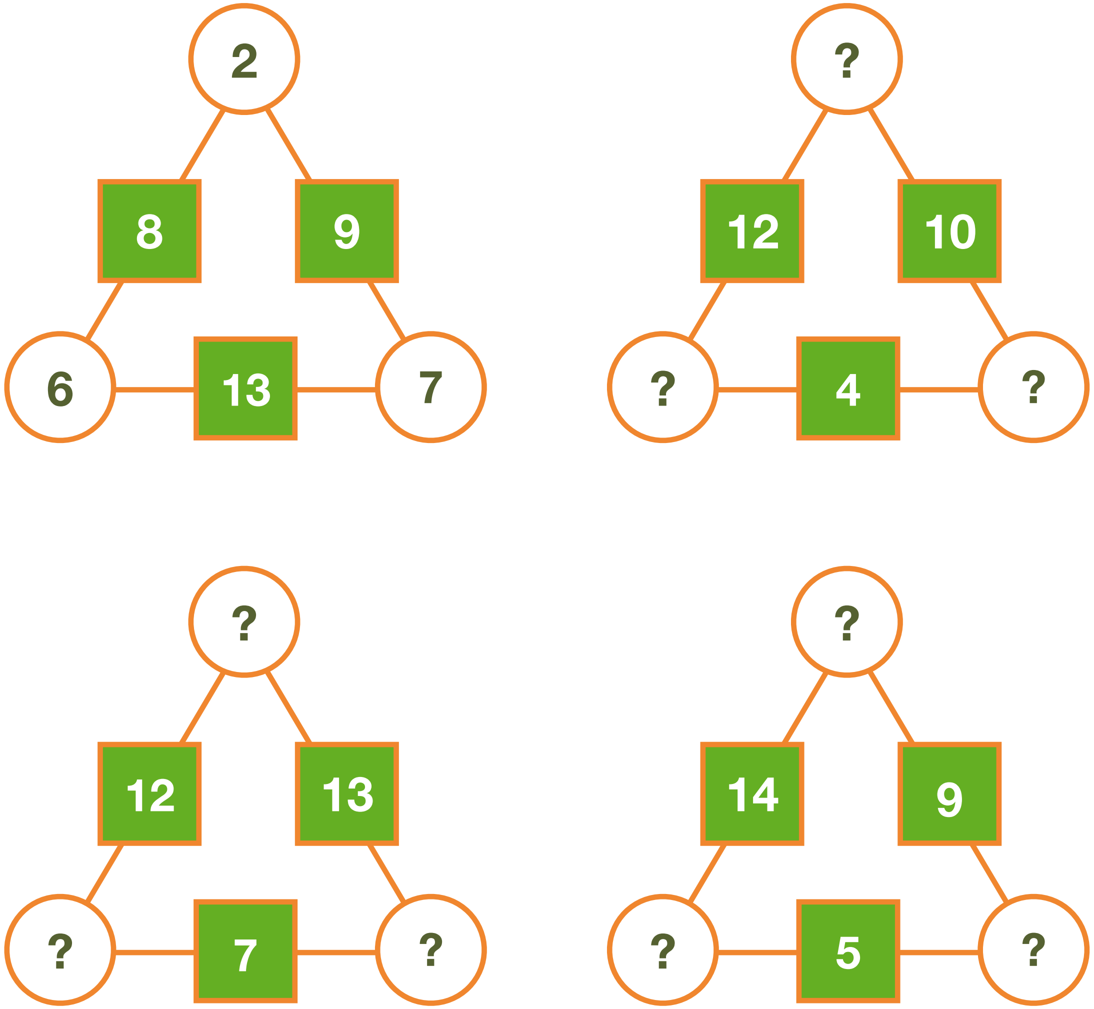
1. the first digit from the left (the billions digit) is divisible by 1
2. the number formed from the first 2 digits from the left is divisible by 2
3. the number formed from the first 3 digits from the left is divisible by 3
4. the number formed from the first 4 digits from the left is divisible by 4
5. the number formed from the first 5 digits from the left is divisible by 5
6. the number formed from the first 6 digits from the left is divisible by 6
7. the number formed from the first 7 digits from the left is divisible by 7
8. the number formed from the first 8 digits from the left is divisible by 8
9. the number formed from the first 9 digits from the left is divisible by 9
10. the number itself is divisible by 10

Arithmagons

Can you understand how the values of the circles determine the value of the squares?

Can you fill in the circles irrespective of what is in the squares?

Describe and explain what you have discovered.



Big Powers

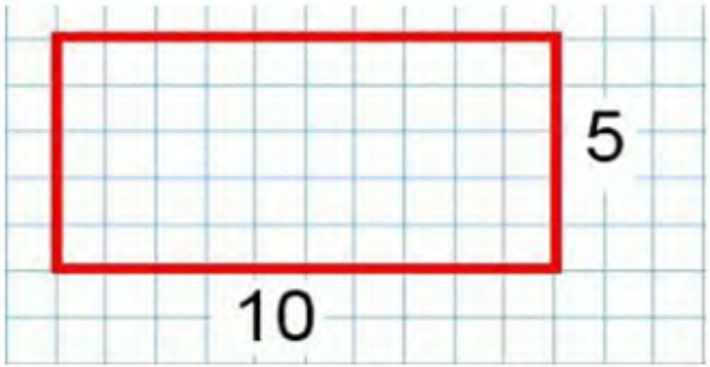
Is the number

$$3^{444} + 4^{333}$$

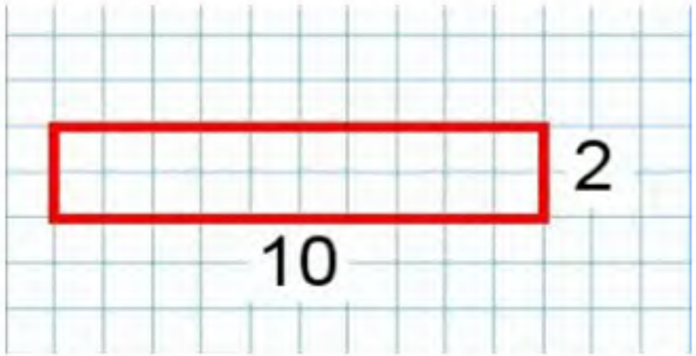
divisible by 5?

Can They Be Equal?

Can you find a rectangle where the perimeter and the area have the same numerical value?



Area = 50 units^2
Perimeter = 30 units



Area = 20 units^2
Perimeter = 24 units

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Chances Are

Are you willing to take your chances with any of these games?

Which one has the best odds of winning?

To win, spin a coin and get 12 heads in a row!

Roll a 6 on our ten-sided die four times in a row to win!

Pick my favourite 4 of these 10 pictures and put them in order to win

Throw five dice and get five sixes, and you win!

These are my seven favourite plants. Put them in the right order to win.

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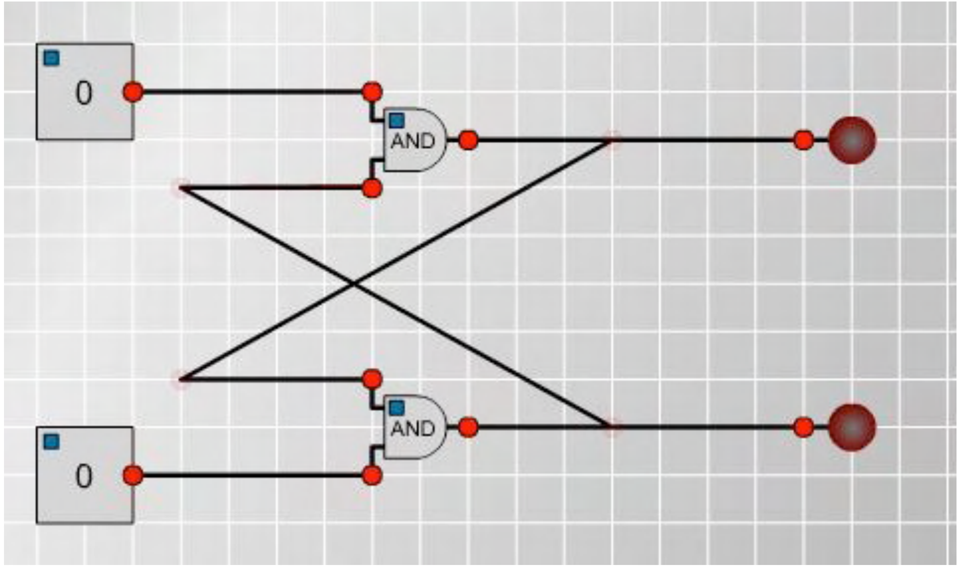
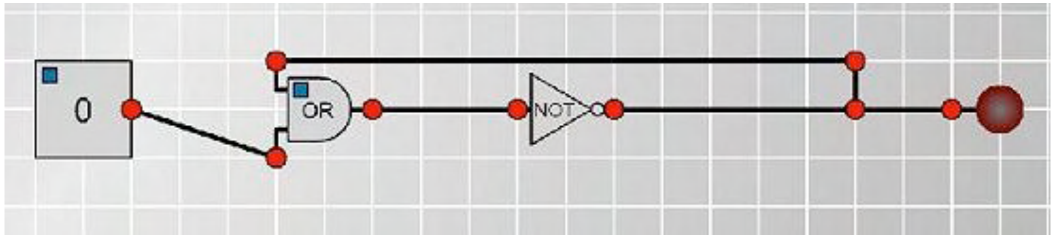
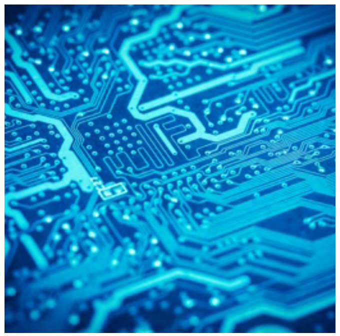
Circular Circuitry

What will happen when you switch on these circuits?

What will happen if you change the gates to different types?



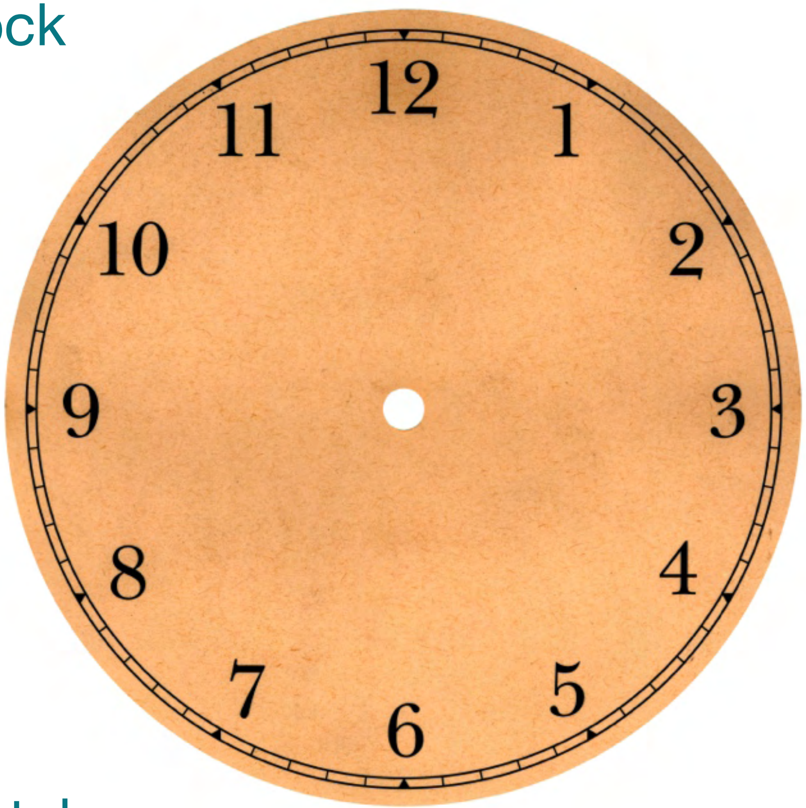




Clock Face

Can you draw a straight line across the centre of a clock face so that the numbers on both sides of the line have the same total?

Can you draw two lines (like the hands of the clock) to divide the clock face so that the total of the numbers on one side of the lines is twice the total on the other side?



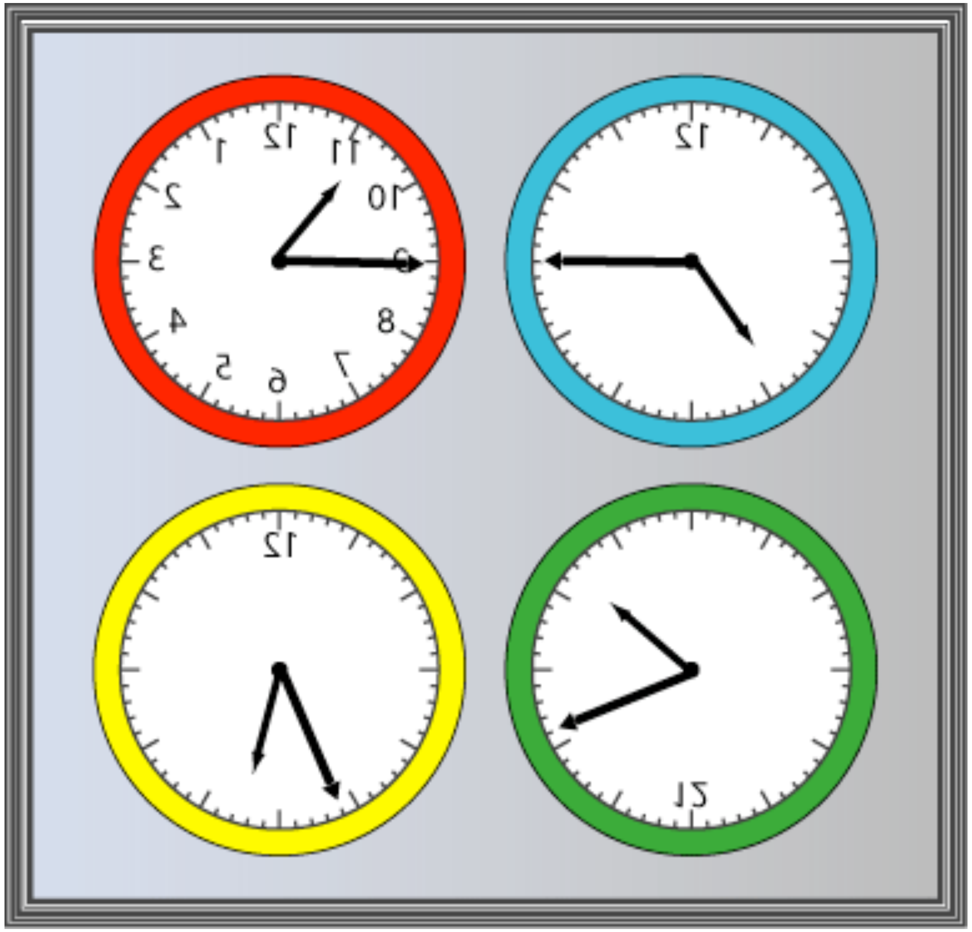
Can you draw two lines so that the numbers on each side add up to a prime number? Can you do this in another way?

Can you find any other interesting ways to group the numbers on a clock face by drawing two lines?

Clocks

These clocks have been reflected in a mirror.

What times do they say?



Coins

A man has five coins in his pocket.

He can make 13 different amounts of money with his coins.

The amounts of money he can make end with one of two possible digits.

He cannot make up exactly 70 pence.

He cannot afford an item costing one pound.

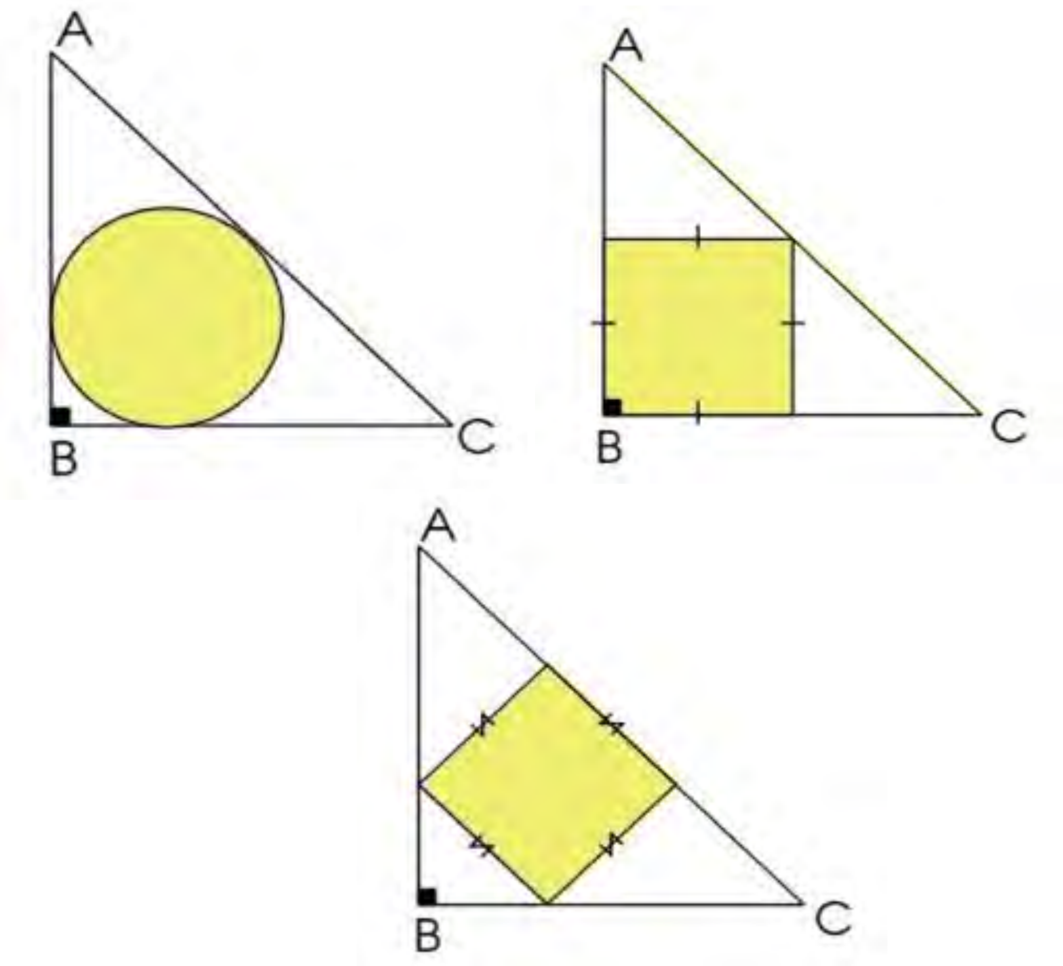
He can make a prime number bigger than ten with his coins.

Can you work out what the coins are?

Compare Areas

ABC is an isosceles right angled triangle.

Which of the inscribed figures has the greatest area?



Consecutive Seven

Can you arrange these numbers into seven subsets each of three numbers so that, when the numbers in each are added together, they make seven consecutive numbers?

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20

Consecutive Sums

10 = 1 + 2 + 3 + 4

11 = 5 + 6

9 = 4 + 5 and 2 + 3 + 4

Some numbers are sums of consecutive numbers.

Can you make all the numbers this way?

Which numbers can be written in more than one way?

12 = 3 + 4 + 5

13 = 6 + 7

14 = 2 + 3 + 4 + 5

Counting Fish

We need to estimate the fish population in a lake.

We catch 40 fish and tag them so that they can be identified if caught again.

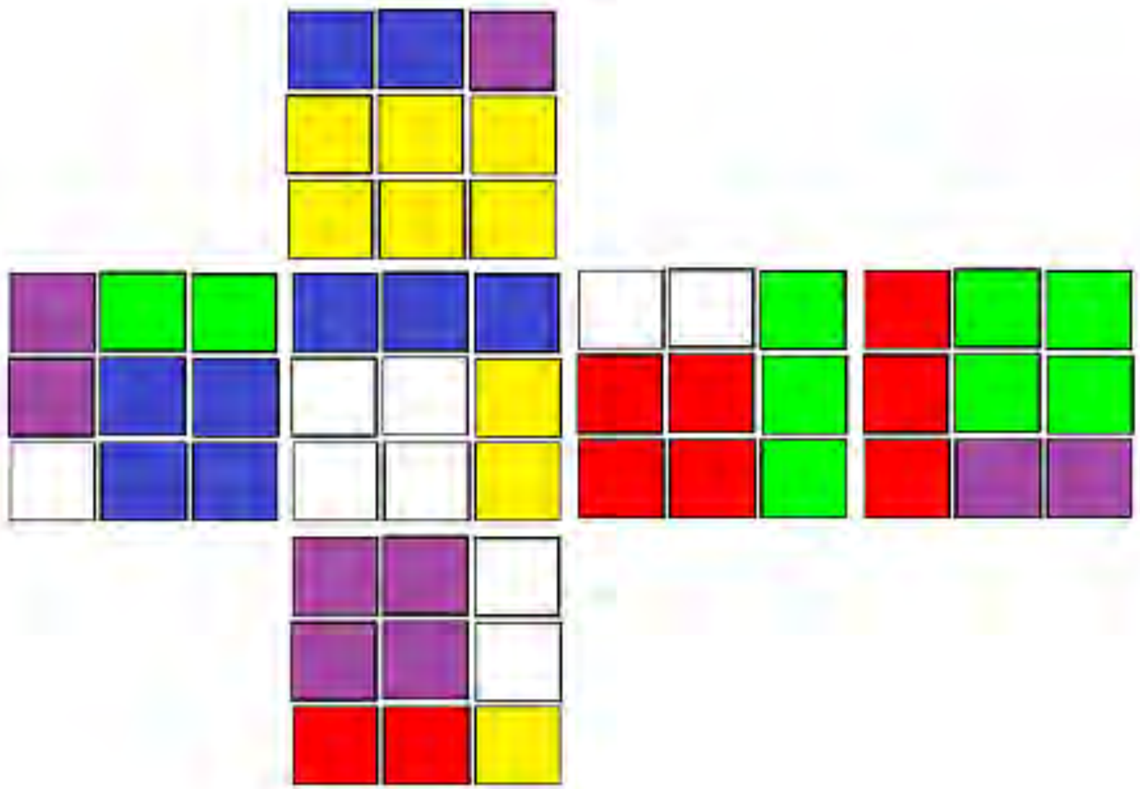
The fish are then released and one week later we again catch 40 fish and look to see how many are tagged.

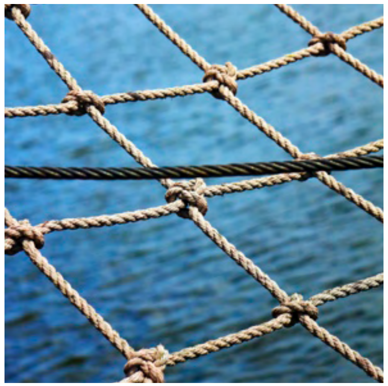
How could this help us come up with a figure for the fish population in the lake?

Cubic Net

This is the net of a Rubik's cube which is just three twists away from completion.

Can you work out how to complete the net?

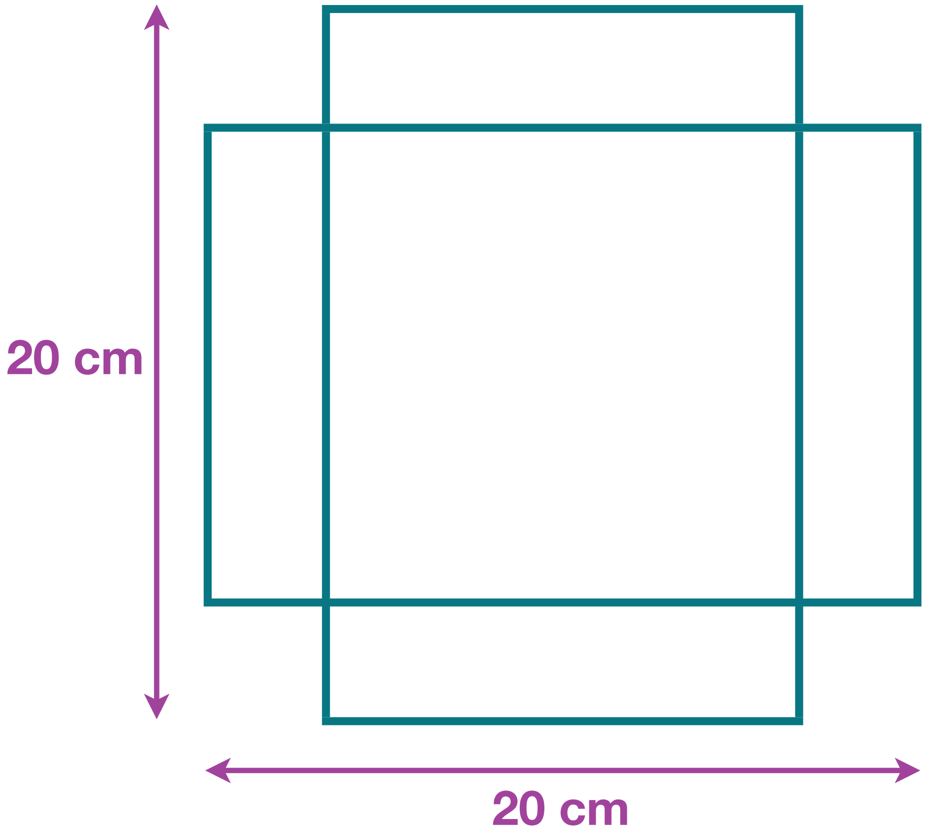




Cuboid Challenge

You can make an open box from a 20cm by 20cm piece of card by cutting out four squares and folding the flaps.

What's the biggest volume of box you can make in this way?



Cuboids

Find a cuboid (with edges of whole number lengths) that has a surface area of exactly 100 square units.

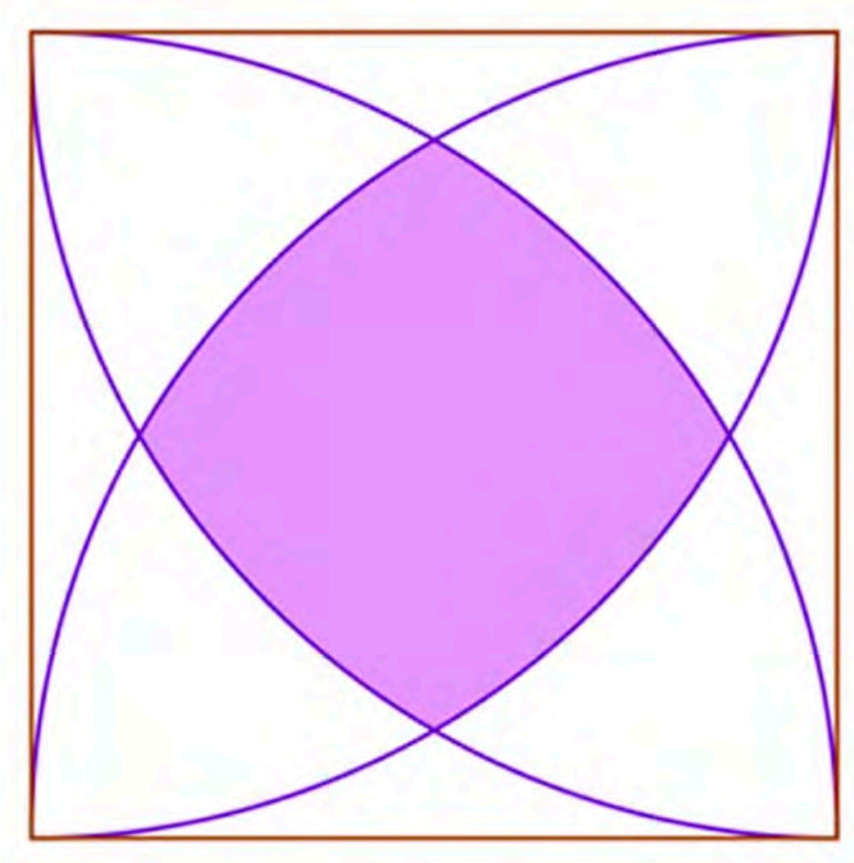
Is there more than one?

Can you find them all?

Can you provide a convincing argument that you have found them all?

Curved Square

A square of side length 1 has a circle of radius 1 drawn from each of its corners, as in the diagram. The circles intersect inside the square at four points, to create the shaded region.

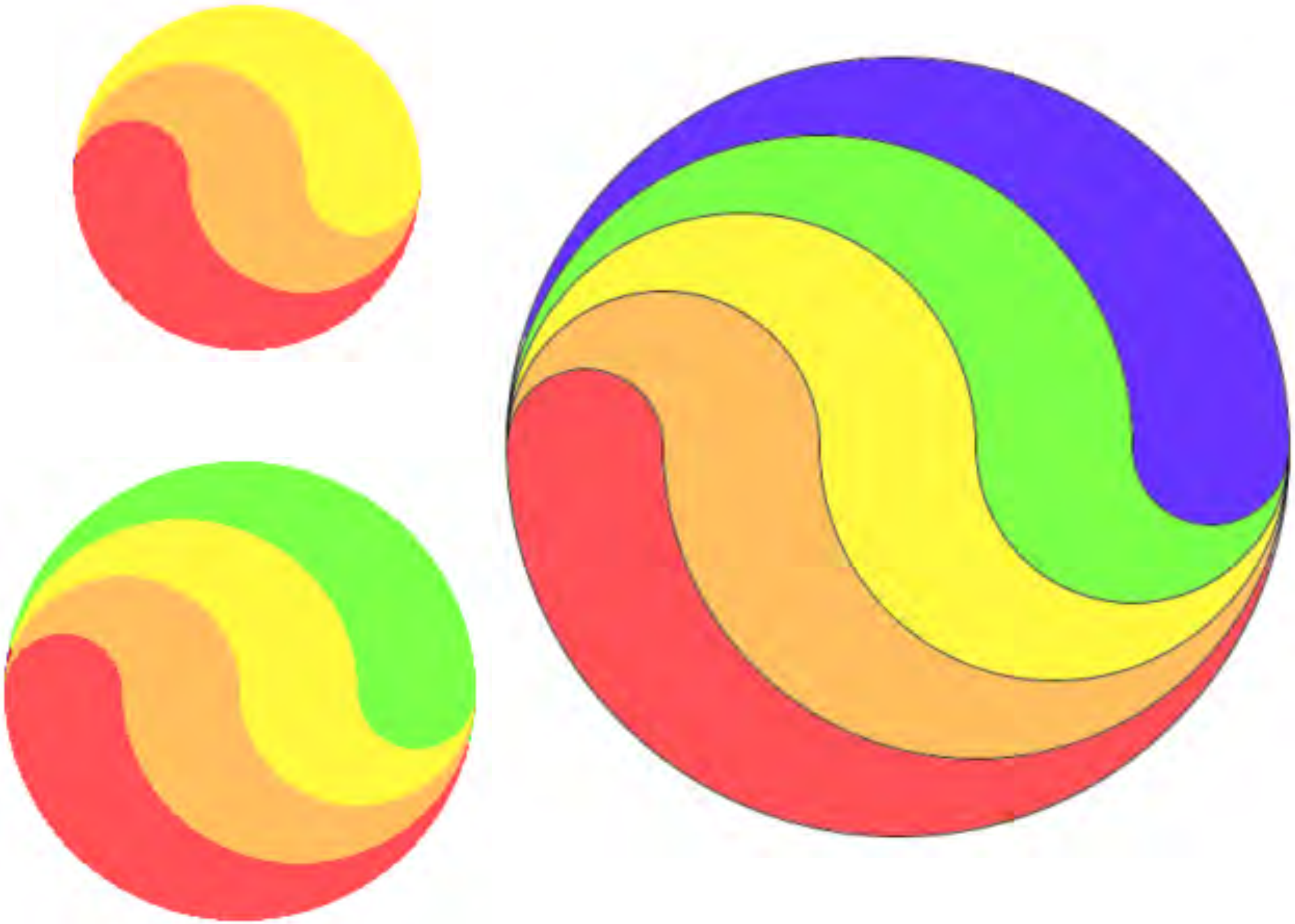


What is the exact area of the shaded region?

Curvy Areas

What proportion of each shape is shaded in each colour?

Can you generalise?



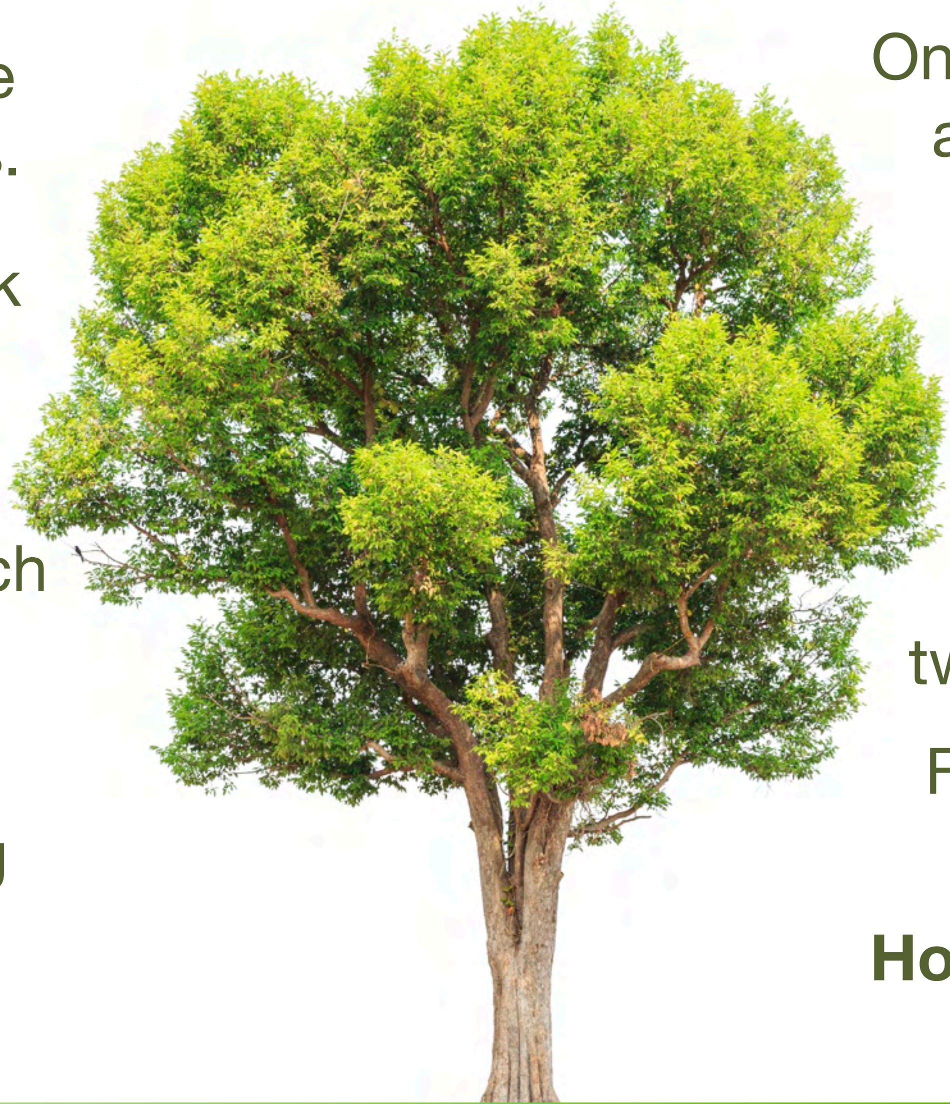
The Deca Tree

The deca tree has 10 trunks.

On each trunk there are 10 branches.

On each branch there are 10 twigs.

On each twig there are 10 leaves.



One day a woodcutter came along and cut down one trunk from the tree.

Then he cut off one branch from another trunk of the tree.

He then cut off one twig from another branch.

Finally he pulled one leaf from another twig.

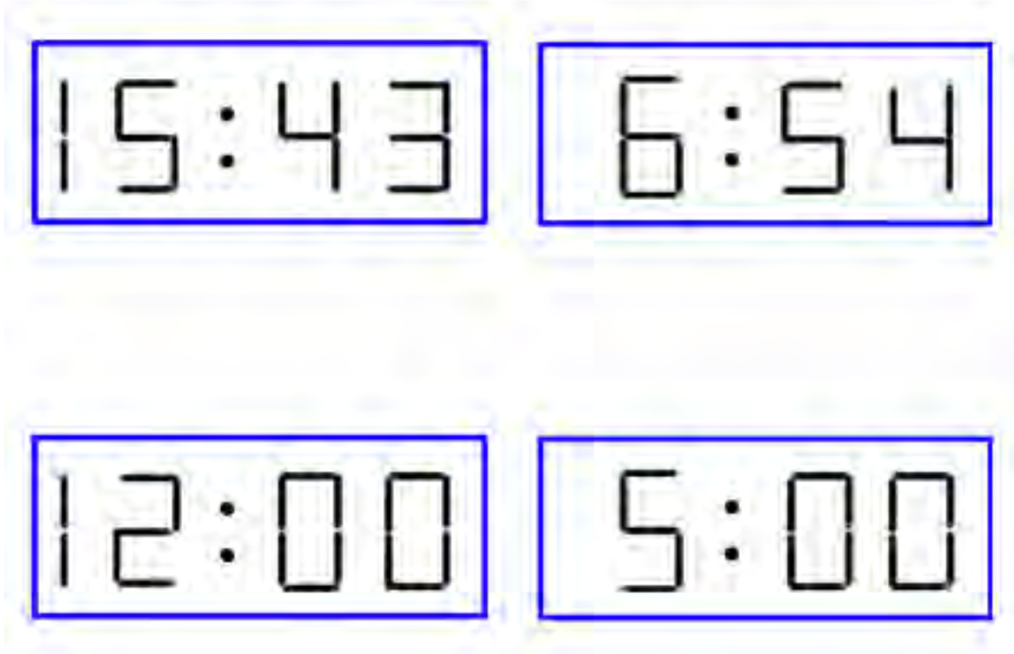
How many leaves were left on the tree?

Decimal Time

In France in 1793 decimal time was introduced, then abandoned only two years later.

Look at these pairs of times. The ones on the left are our time, and the clocks on the left show the corresponding French decimal time.

Can you decipher how French decimal time works?



Dicey Operations

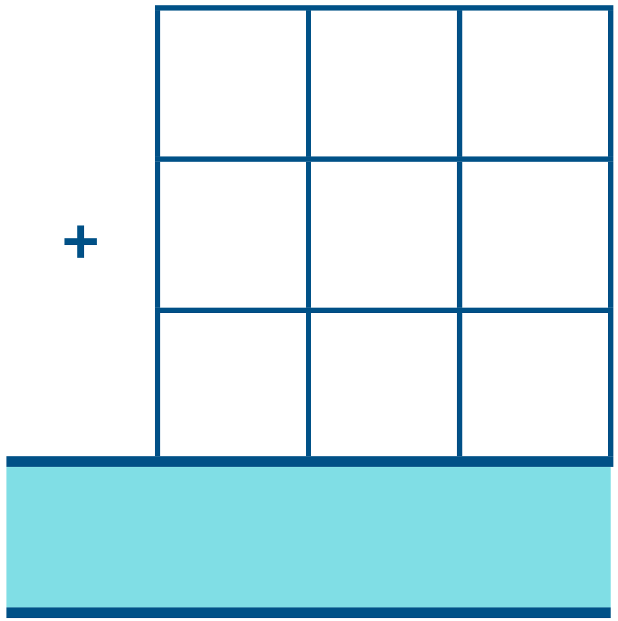
Find a partner and a die (preferably 0 - 9 but if you don’t have one you can use a 1 - 6 die).

Each of you draw an addition grid like the one on the right.

Take turns to throw the die and decide which of your cells to fill in.

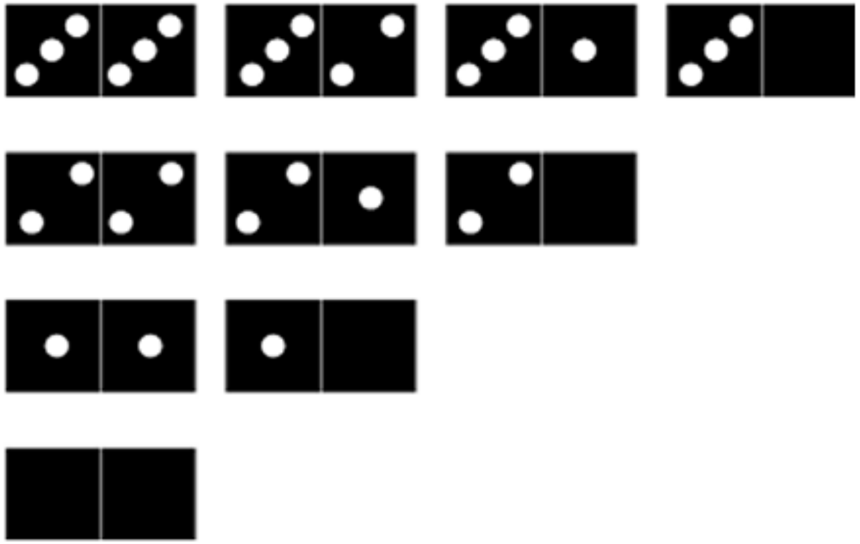
Throw the die nine times each until all the cells are full.

Whoever has the sum closest to 1000 wins.

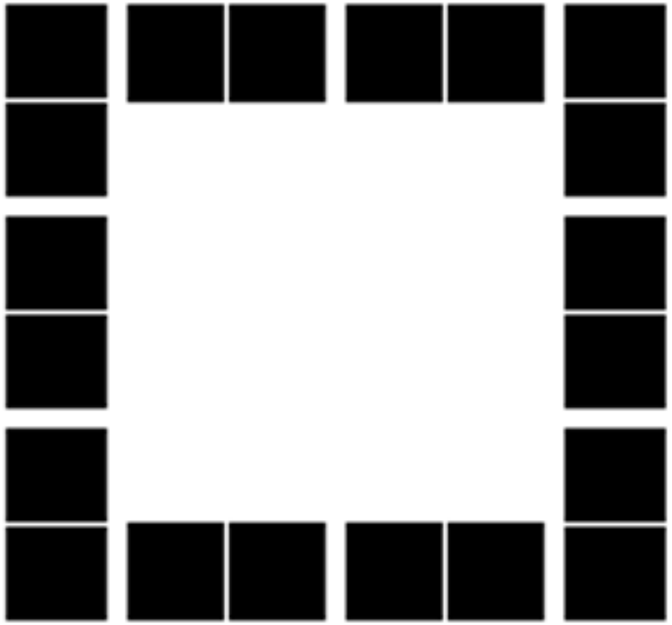


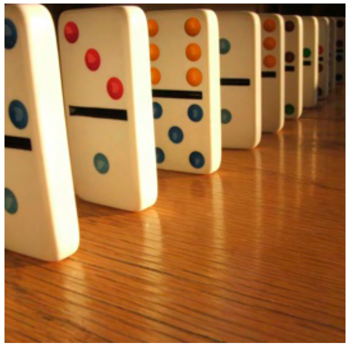
Domino Square

These are the ‘double-3 down’ dominoes.



Use these dominoes to make this square so that each side has eight dots.





Dotty Six

Here's a game to play with a friend, a 3 by 3 grid and a six-sided dice.

Take turns to throw a dice, then draw that many dots in one of the boxes on the grid. You can't split them up and a box can't contain more than six dots.

The person who completes a line of three sixes wins!

Dozens

What is the largest possible five-digit number divisible by 12 that you can make from the digits 1, 3, 4, 5 and one more digit?

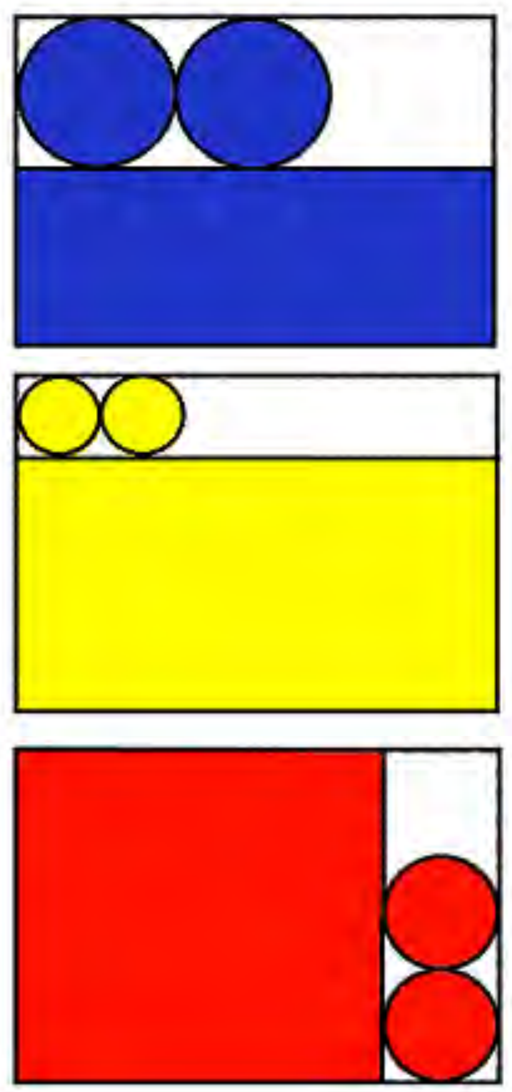
Efficient Cutting

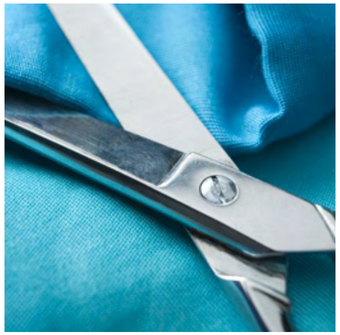
A cylindrical container can be made by using two circles for the ends and a rectangle which wraps round to form the body.

To make cylinders of varying sizes, the three pieces can be cut from a single rectangular sheet in several ways. Some examples are shown here.

Using a single sheet of A4 paper, make the cylinder with the largest volume. The cylinder must be closed off with a circle at each end.

What are its dimensions?





Eggs in Baskets

There are three baskets, a brown one, a red one and a pink one, holding a total of ten eggs.

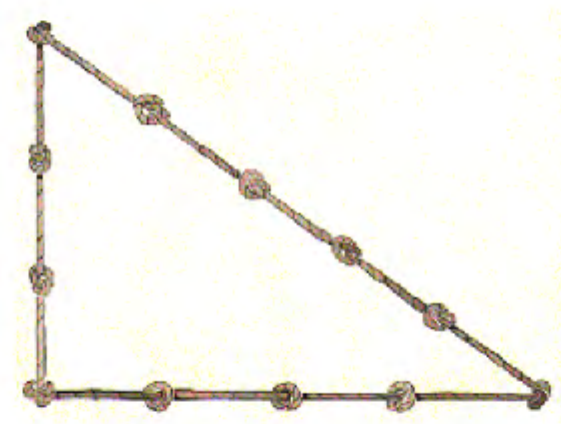
The brown basket has one more egg in it than the red basket.

The red basket has three fewer eggs than the pink basket.

How many eggs are in each basket?

Egyptian Rope

The ancient Egyptians were said to make right-angled triangles using a rope which was knotted to make 12 equal sections.



If you have a rope knotted like this, what other triangles can you make? (You must have a knot at each corner.)

What regular shapes can you make - that is, shapes with equal sides and equal angles?

Elevenses

In this grid, how many pairs of numbers can you find that add up to a multiple of 11?

Could you convince someone that you've found them all?

9	46	79	13
64	90	2	97
25	31	20	22
4	52	55	7

A Bowl of Fruit

Half the pieces of fruit in the bowl are apples.

There are also three oranges, two pears and a banana.

How many apples are there in the bowl?

Giants

Which is bigger: 9^{10} or 10^{9} ?

Can you use a calculator to compare 99^{100} and 100^{99} ?

Work out which is bigger out of 999^{1000} and 1000^{999}.

Got It!

This is a game for two players.

Start with the target number of **23**.

The first player chooses a whole number from 1 to 4.

Players take turns to add a whole number from 1 to 4 to the running total.

The player who hits the target of 23 wins the game.

Can you find a winning strategy?
Can you always win?

What happens if you choose a new target number?

What happens if you change the range of numbers you can add?

Can you work out a winning strategy for any target and any range of numbers?

Harmonic Triangles

Can you see what the rules for making the next line are?

Can you work out the next two rows?

Will the second diagonal ($^1/_2$, $^1/_6$, $^1/_{12}$ ...) always contain unit fractions?

Can you prove it?

$$\frac{1}{1}$$

$$\frac{1}{2} \quad \frac{1}{2}$$

$$\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{3}$$

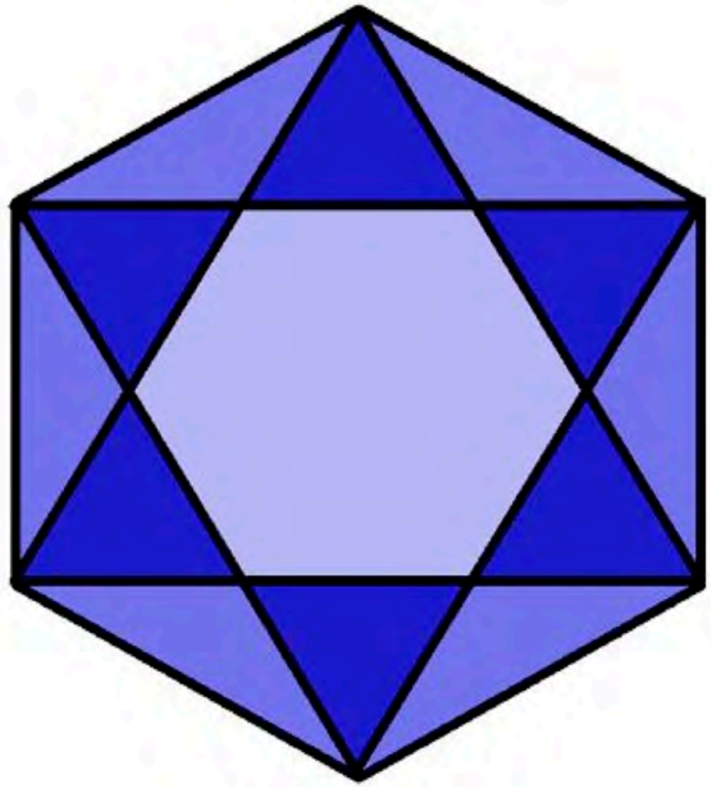
$$\frac{1}{4} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{4}$$

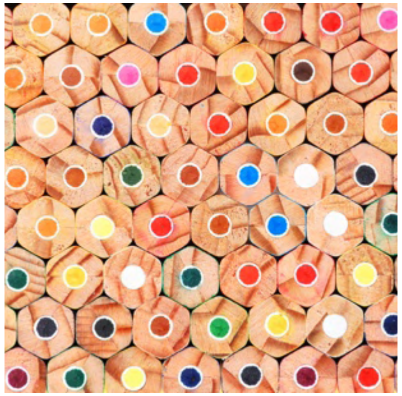
$$\frac{1}{5} \quad \frac{1}{20} \quad \frac{1}{30} \quad \frac{1}{20} \quad \frac{1}{5}$$

$$\frac{1}{6} \quad \frac{1}{30} \quad \frac{1}{60} \quad \frac{1}{60} \quad \frac{1}{30} \quad \frac{1}{6}$$

Hex

Explain how the thirteen pieces making up the regular hexagon shown in this diagram can be reassembled to form three smaller regular hexagons congruent to each other.





How Old Am I?

On my last birthday, a friend said to me:

“In 15 years’ time, your age will be the square of your age 15 years ago!”

Can you work out how old I am?

Hundred Square

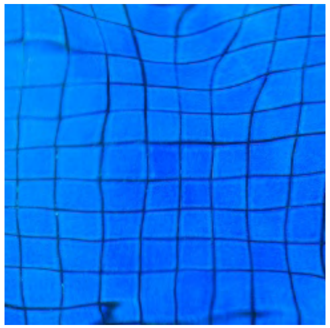
A hundred square has been printed on both sides of a piece of paper.

One square is directly behind the other.

What is on the back of 100? 58? 23? 19?

Can you see a pattern?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



In a Box

There are **two red** ribbons and **four blue** ribbons in a box.

We take two ribbons out without looking.

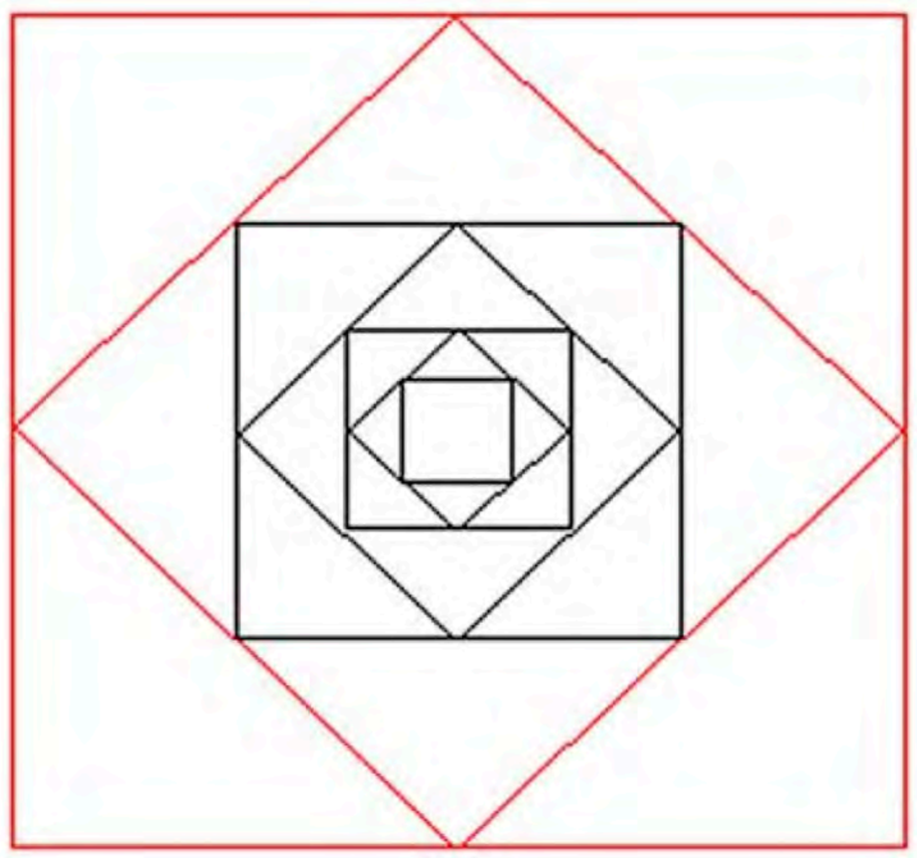
You win if they are **the same colour**, and I win if they are **different colours.**

Is it a fair game?

Inside Seven Squares

Seven squares are set inside each other. The centre points of each side of the outer square are joined to make a smaller square inside it, and so on.

The centre square has the area of one square unit. What is the total area of the four outside triangles that are outlined in red?



Isosceles Triangles

On a square grid, draw some isosceles triangles with an area of $9cm^2$ and a vertex at (20,20).

If all the vertices have whole number co-ordinates, how many different triangles is it possible to draw?

Can you explain how you know that you have found them all?

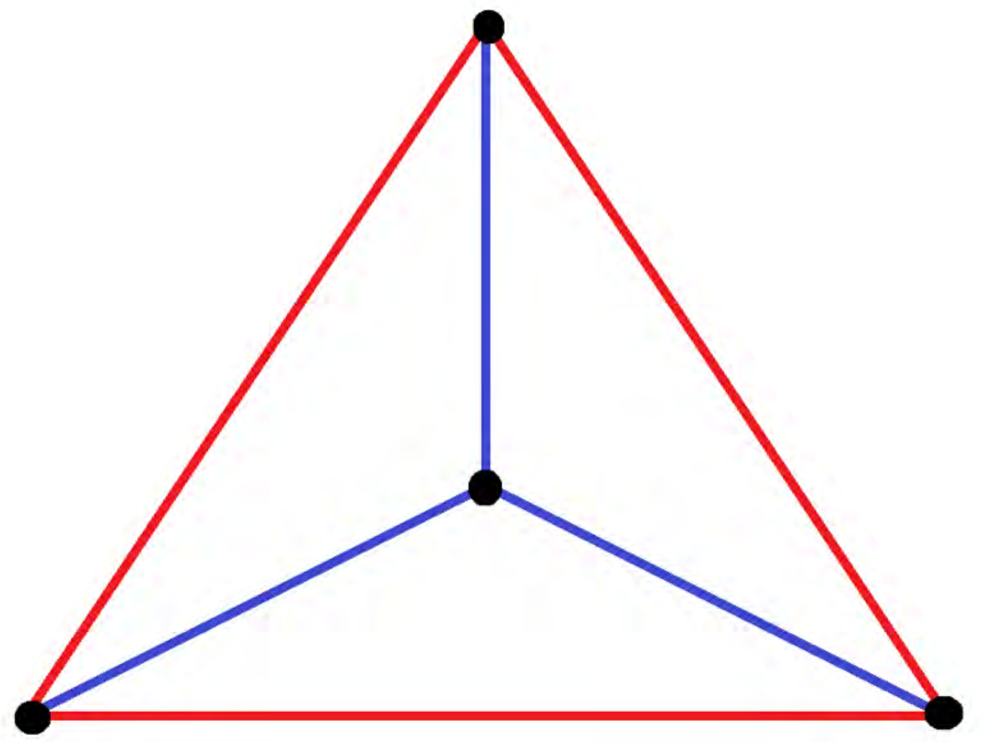
Keep Your Distance

Mark four points on a flat surface so that there are only two different distances between them.

One arrangement is shown.

How many more can you find?

Are you sure that you have them all?



Largest Product

3 + 3 + 4 = 10
3 x 3 x 4 = 36

3.3 + 6.7 = 10
3.3 x 6.7 = 22.11

What is the greatest product that can be made from numbers that add up to 10?

5 + 5 = 10
5 x 5 = 25

1 + 9 = 10
1 x 9 = 9

1 + 2 + 3 + 4 = 10
1 x 2 x 3 x 4 = 24

Legs Eleven

Take a four-digit number: 3527.

Move the first digit to the back of the queue and move the rest along, giving 5273.

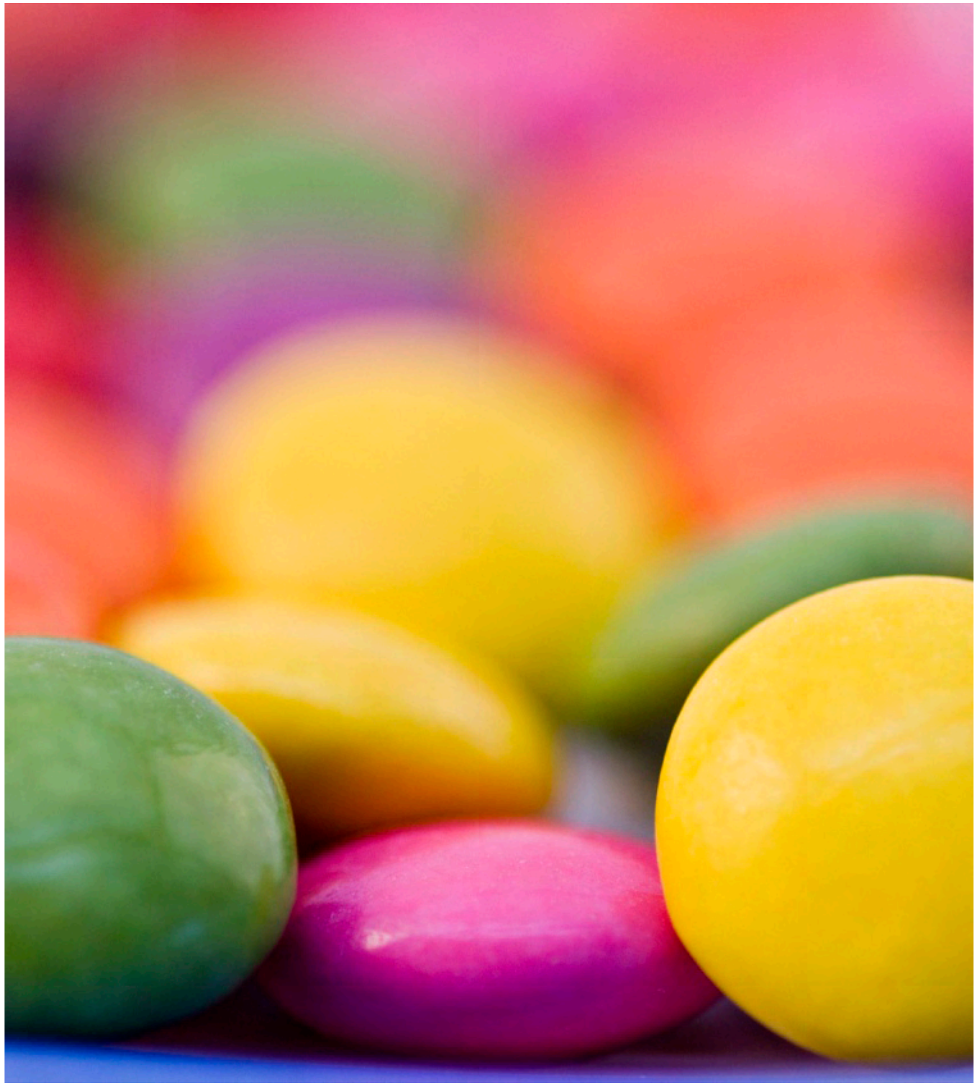
Now add your two numbers.

Now try a few other four-digit numbers. What do all your answers have in common?

Why?

Does it work for two-digit, three-digit, five-digit, 38-digit ... numbers?

M, M and M



I have five numbers.

Their mean is 4.

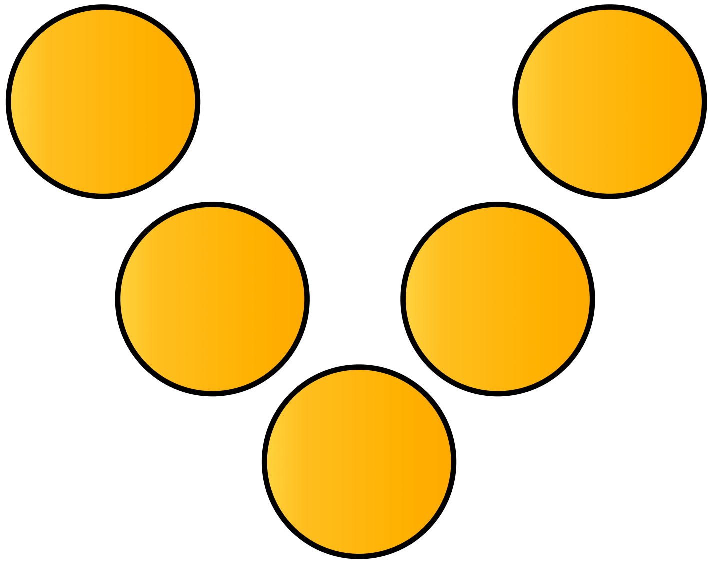
Their median is 3.

Their mode is 3.

Can you find *all* the different sets of five positive whole numbers that satisfy these conditions?

Magic Vs

Place each of the numbers 1 to 5 in the V shape so that the two arms of the V have the same total.



How many different possibilities are there?

Can you convince someone that you have all the solutions?

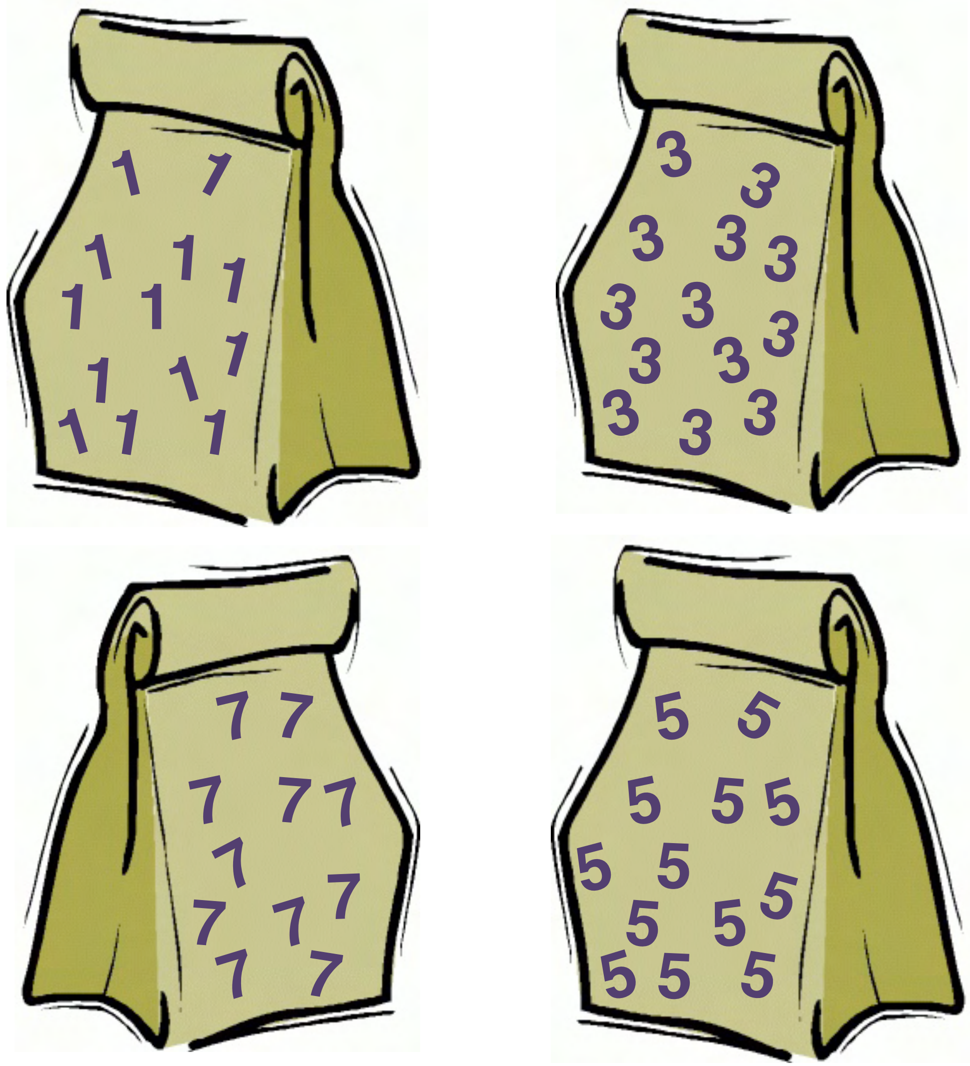
What happens if we use the numbers from 2 to 6? From 12 to 16? From 37 to 41? From 103 to 107?

Investigate the same problem with a V that has arms of length 4.

Make 37

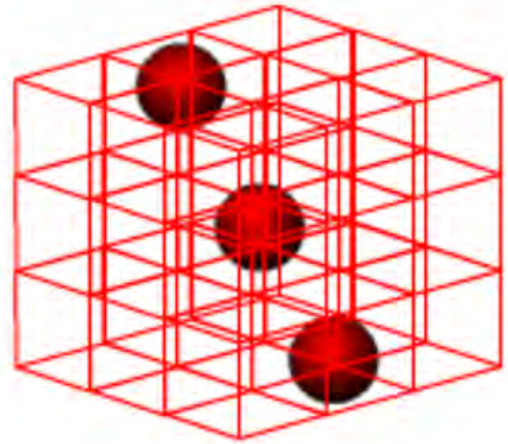
Four bags contain a large number of 1s, 3s, 5s and 7s.

Pick any ten numbers from the bags so that their total is 37.



Marbles in a Box

Imagine a three dimensional version of noughts and crosses where two players take it in turn to place different coloured marbles in a box.



The box is made from 27 transparent unit cubes arranged in a 3-by-3 array.

The object of the game is to complete as many winning lines of three marbles as possible.

How many different ways can you make a winning line?

Mixed Up Socks

Start with three pairs of socks.

Now mix them up so that no mismatched pair is the same as another mismatched pair.

Now try it with four pairs of socks.
Is there more than one way to do it?

Mixing Lemonade

I mixed up some lemonade in two glasses.

The first glass had 60ml of lemon juice and 200ml of water.

The second glass had 100ml of lemon juice and 350ml of water.

Which mixture tasted stronger? How do you know?

Multiplication Square

Take a multiplication square.

Pick any 2 by 2 square and add the numbers on each diagonal. For example, if you take:

32	36
40	45

the numbers along one diagonal add up to 77 and the numbers along the other diagonal add up to 76.

Try a few more examples. What do you notice?

Now pick any 3 by 3 square and add the numbers on each diagonal.

What do you think will happen with a 4 by 4 square?

Can you prove your prediction?

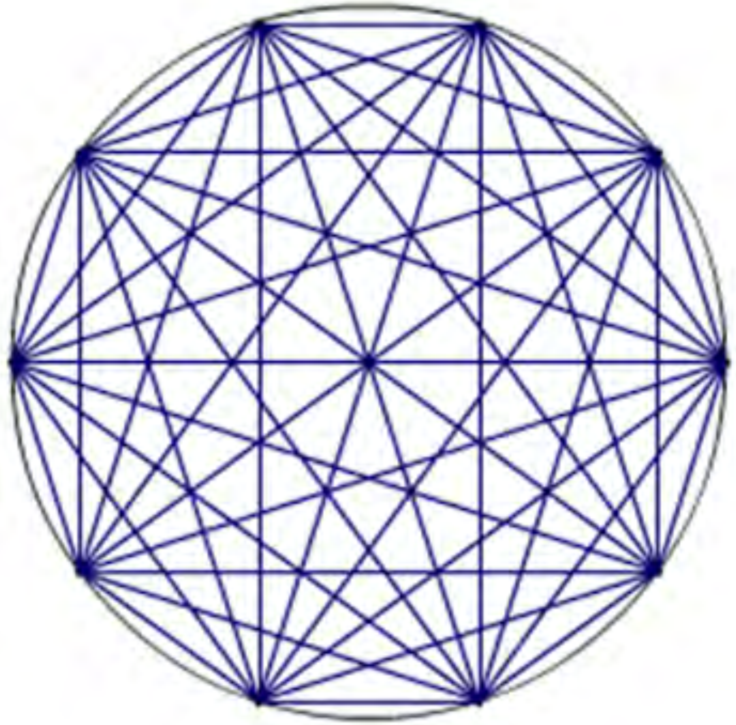
Mystery Matrix

Can you fill in the multiplication square?

The numbers 2 to 12 were used to generate it, with exactly one number used twice.

x						
	32			40		
					49	
			22			
		15				27
			24			
					42	

Mystic Rose



This is a 10 pointed mystic rose. The 10 points are equally spaced around the circle.

How many lines are needed to draw it?

How many lines would you need for a 100 pointed mystic rose?

Negative Dice

I have two identical dice with the numbers

-1, 2, -3, 4, -5, 6

I roll my two dice and work out the total.

Which of the following totals cannot be achieved?

a) 3
b) 7
c) 8

Nine Colours

You have 27 small cubes, 3 each of nine colours.

Can you use all the small cubes to make a 3 by 3 by 3 cube so that each face of the bigger cube contains one of each colour?

Noah

Noah saw 12 legs walk by into the Ark.

How many creatures could he have seen?

How many different answers can you find?

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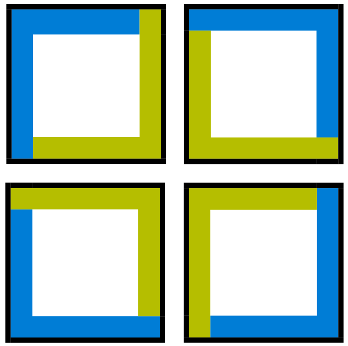
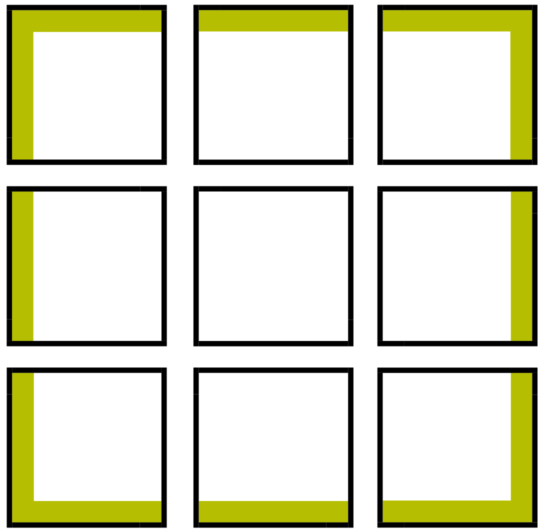
On the Edge



Four tiles can be painted and arranged so that the edge of the large square is blue (as shown) or green.

Can you paint and arrange nine tiles so the edge of the large square can be green, blue or yellow?

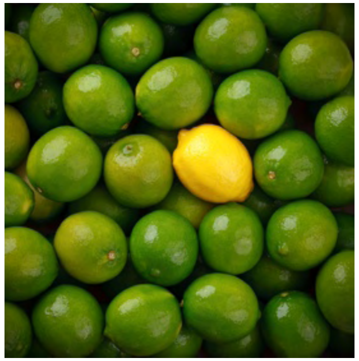


One of Thirty Six

Can you find the chosen number from this square using the clues below?

1. The number is odd
2. It is a multiple of three
3. It is smaller than 7 x 4
4. Its tens digit is even
5. It is the greater of the two possibilities

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36



Online

This is a game for two players.

To play the game, take words alternately.

You win if you get all the occurrences of the same letter (e.g. AN, ON and LINE contain all occurrences of the letter N).

Can you devise a strategy so that you never lose?

Can you explain your strategy?

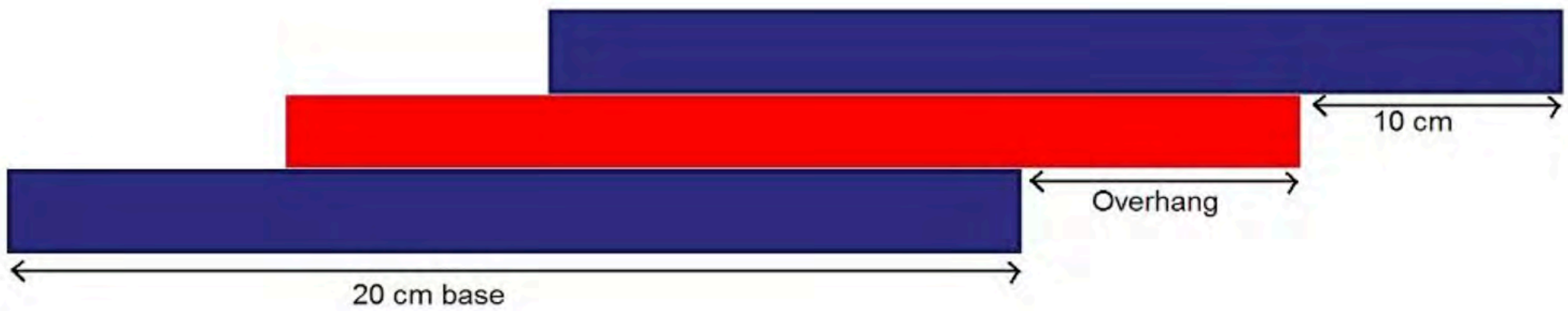
EAT	AN	LAF
IT	LINE	IF
LOT	ON	FOE

Overarch

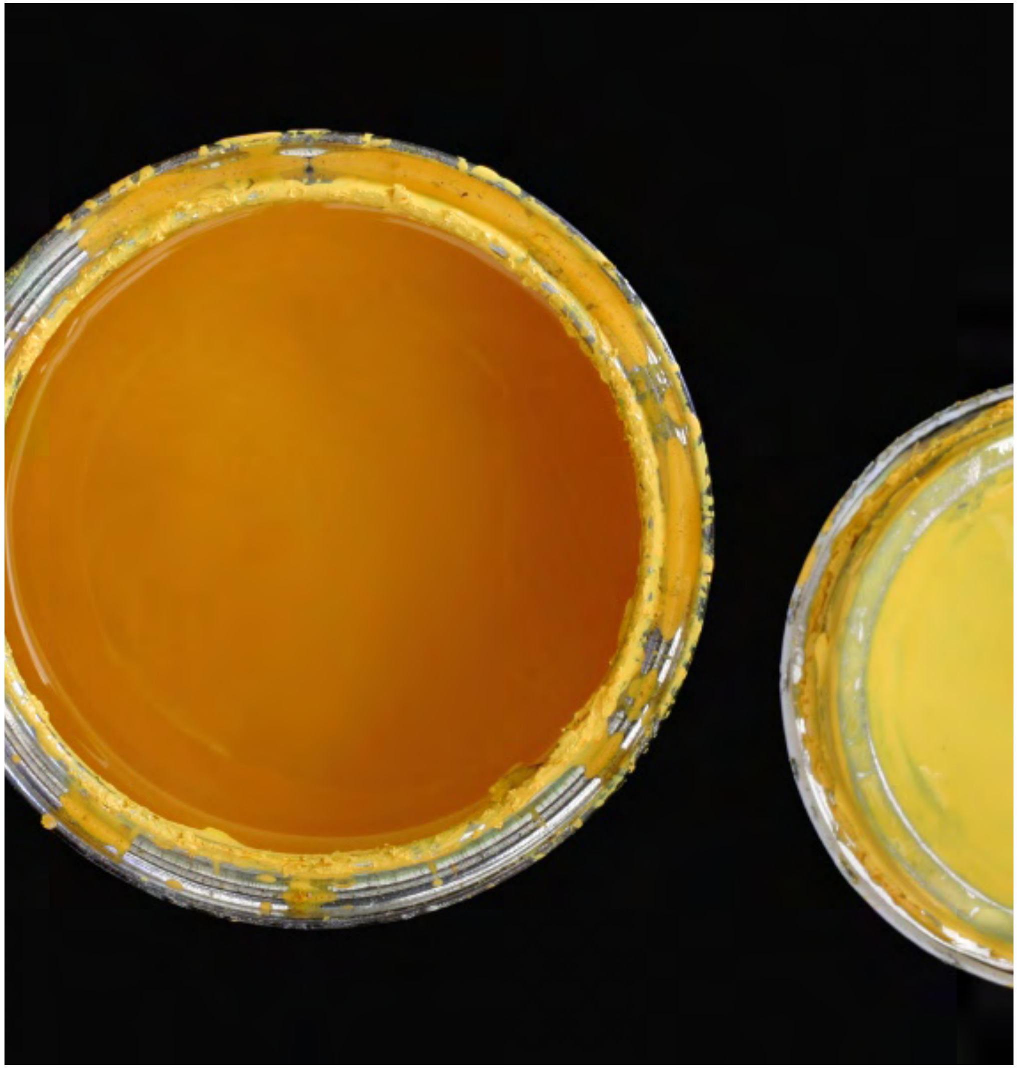
Three uniform square tiles of side 20cm are balanced as shown.

The red tile overhangs as far as possible without toppling over.

How big is the overhang?



Painted Cube



Imagine a large cube made up from 27 small blue cubes.

Imagine dipping the large cube into a pot of orange paint so the whole outer surface is covered, and then breaking the cube into its small cubes.

What colours will the faces of the 27 small cubes be now?

Pair Products

Choose four consecutive whole numbers, for example, 4, 5, 6 and 7.

Multiply the first and last numbers together.

Multiply the middle pair together.

Choose different sets of four whole consecutive numbers and do the same.

What do you notice?

Try sets of five consecutive whole numbers and do the same.

What do you notice now?

What happens when you take *n* consecutive whole numbers?

Explain your findings.

Peaches Today, Peaches Tomorrow

A monkey has 75 peaches. Each day, he kept a fraction of his peaches, gave the rest away, and then ate one.

These are the fractions he decided to **keep**:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{3}{5} \quad \frac{5}{6} \quad \frac{11}{15}$$

In what order did he use the fractions so that he was left with just one peach at the end?

Pick’s Theorem

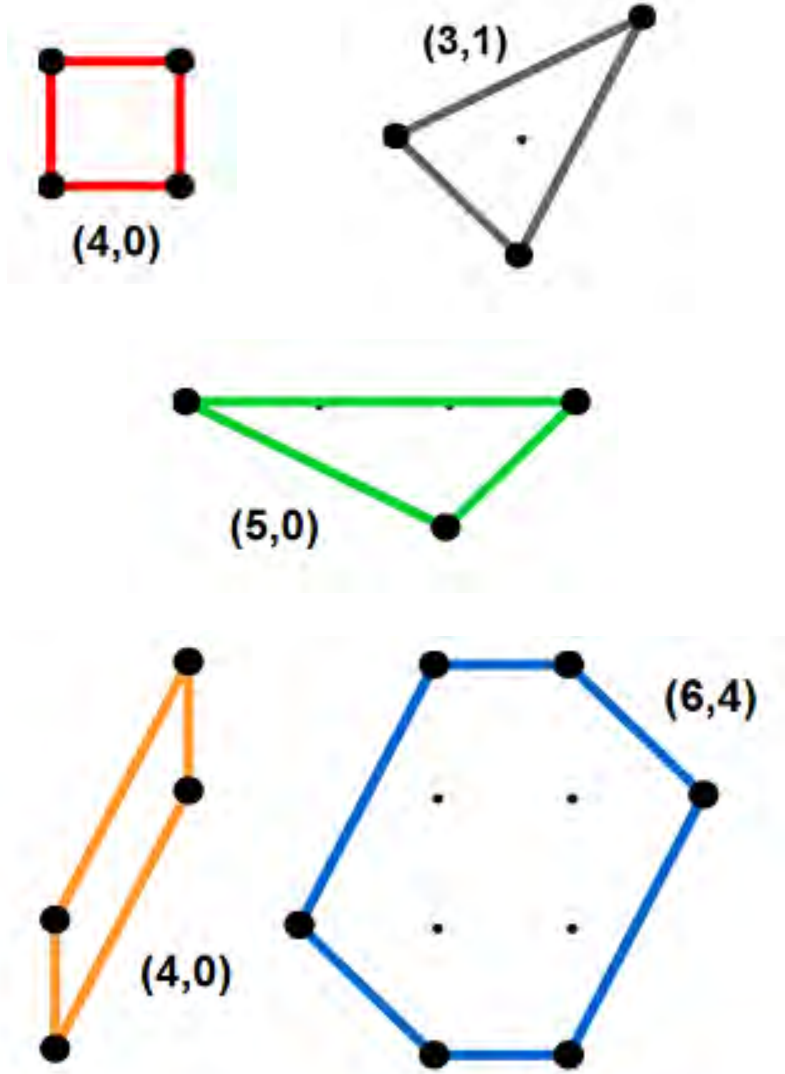
When the dots on square dotty paper are joined by straight lines the resulting figures have dots on their perimeter (p) and often internal (i) ones as well.

Each figure can be described as (p,i).
How many different figures can be described as (4,0)?

Each figure always encloses an area (A).

Draw more figures: tabulate the information about their perimeter points (p), interior points (i) and areas (A).

Can you find a relationship between all these three variables (p, i, A)?



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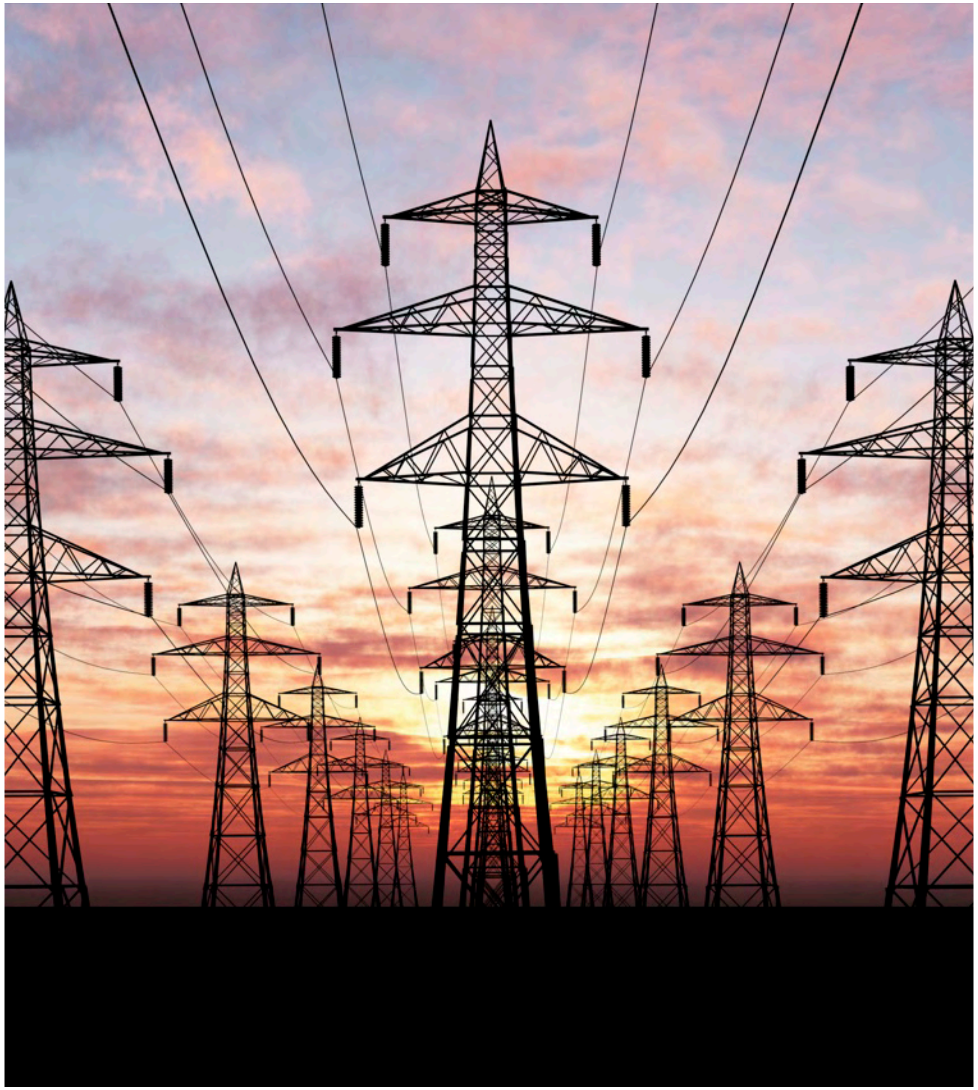
Power Mad!



Can you find convincing arguments that explain why all the statements below are true?

a) $2^1 + 3^1, 2^3 + 3^3, 2^5 + 3^5, \ldots, 2^{99} + 3^{99}$ are all multiples of 5.

b) $1^{99} + 2^{99} + 3^{99} + 4^{99}$ is a multiple of 5.

c) $1^x + 2^x + 3^x + 4^x + 5^x$ is a multiple of 5 when x is odd

Reflecting Squarely

In how many ways can you fit all three pieces together to make shapes with line symmetry?

There are more than five solutions to this problem.

How many can you find?

Can you find them all?

Route to Infinity

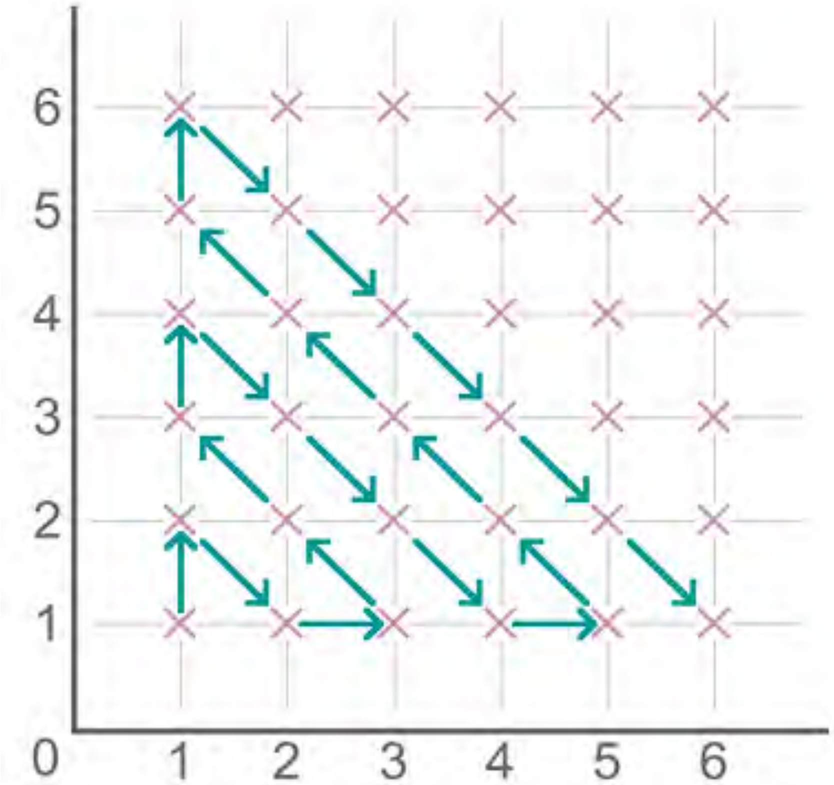
Take some time to look at the route the arrows follow in the diagram.

Try to describe their path.

Will the route pass through the point (18,17)?

If so, which point will be visited next?

Through how many points does the route pass before it reaches the point (9,4)?



Searching for Mean(ing)

Imagine you have a large supply of 3kg and 8kg weights.

Four 3kg weights and one 8kg weight have an average weight of 4kg.

If you had other combinations of the 3kg and 8kg weights, what other whole number averages could you make?

Sets of Numbers

How many different sets of numbers with at least four members can you find in the numbers in this box?

For example, one set could be multiples of four, another set could be odd numbers.

2 91 27 51 124 8 36 39 3 17 13 119 56 121 143 136 64 49

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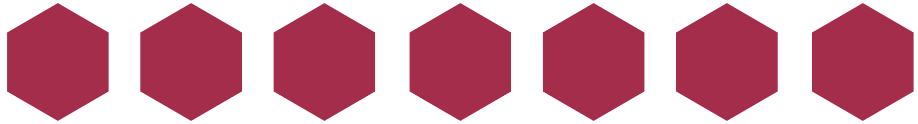
Seven Flipped

You have seven hexagonal-shaped mats, each with one side red and one side blue.



Starting red side up, these mats all have to be turned over - but you can only turn over exactly three at a time.

What is the smallest number of moves you can do this in?

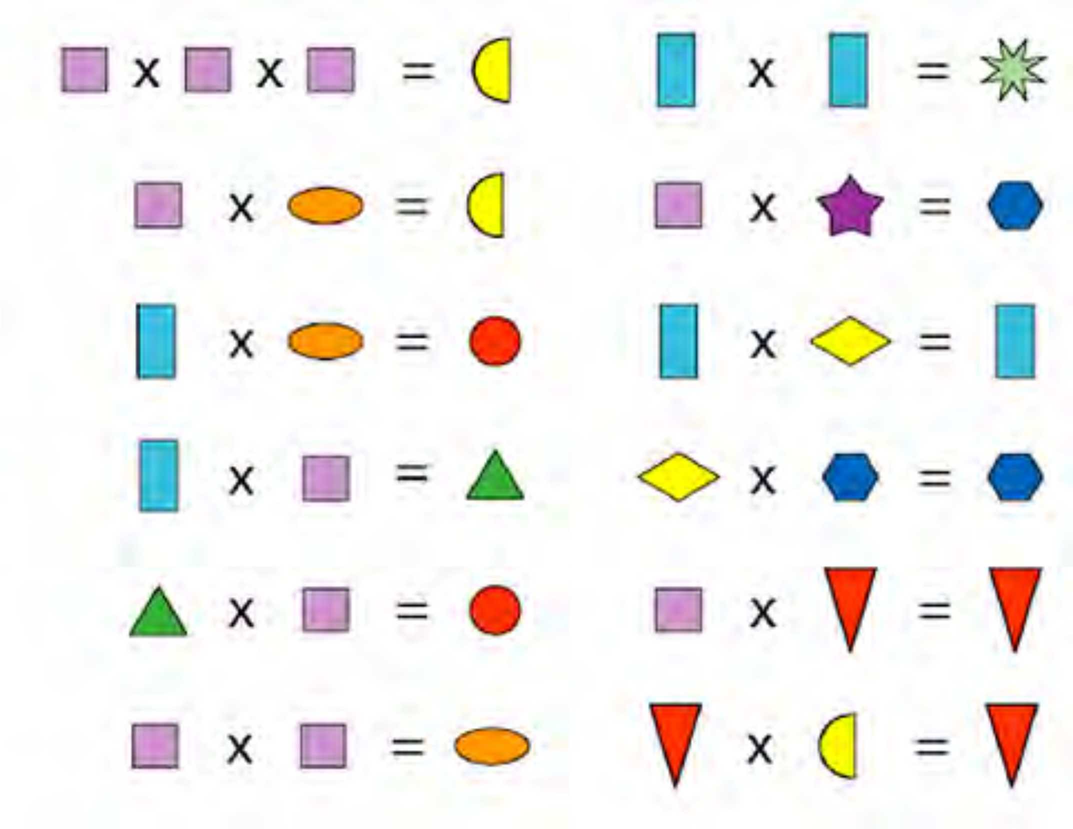
Try with other numbers of mats. Do you notice any patterns in your findings?
Can you explain why these patterns occur?

Shape Times Shape

The coloured shapes stand for eleven of the numbers from 0 to 12.

Each shape is a different number.

Can you work out what they are?



Special Numbers

I'm thinking of a two digit number.

My number is special because adding the **sum** of its digits to the **product** of its digits gives me my original number.

Can you discover my special number?

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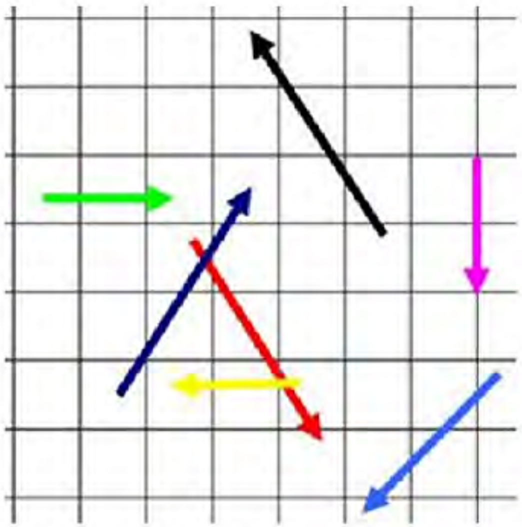
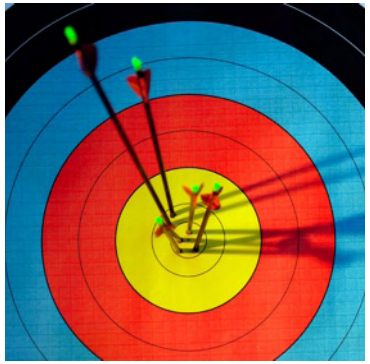
Spotting the Loophole



Each of these grids contains a jumble of vectors with whole number components.

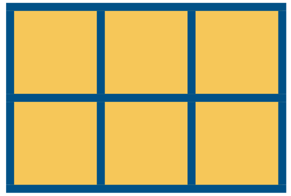
In each case, can you visualise which of the vectors sum to zero?

Can you prove it?

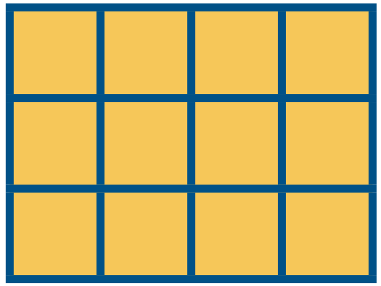


Squares in Rectangles

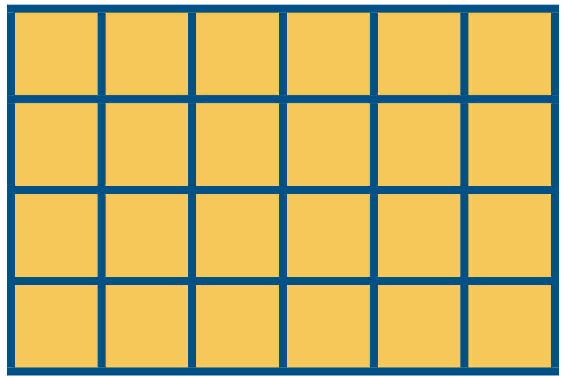
A 2 by 3 rectangle contains 8 squares.



A 3 by 4 rectangle contains 20 squares.



A 4 by 6 rectangle contains 50 squares.

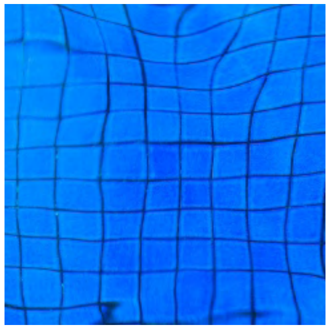


What size rectangle contains exactly 100 squares?

Is there more than one?

Can you find them all?

Can you prove there are no more?

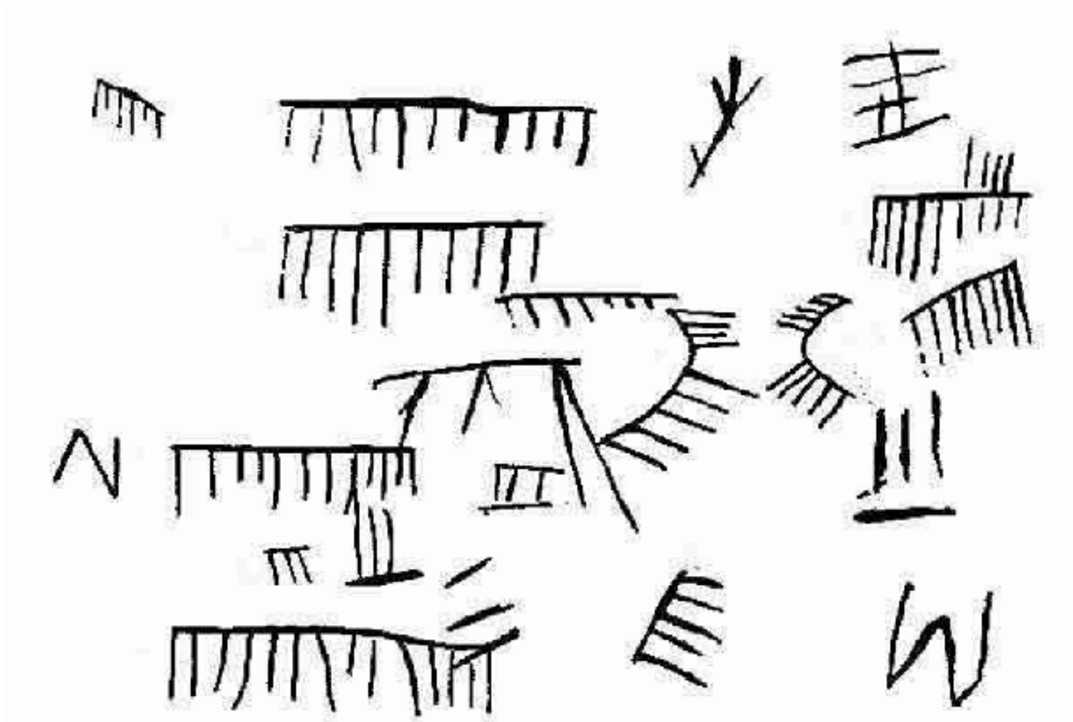


Stone Age Counting

Could these drawings represent counting things?

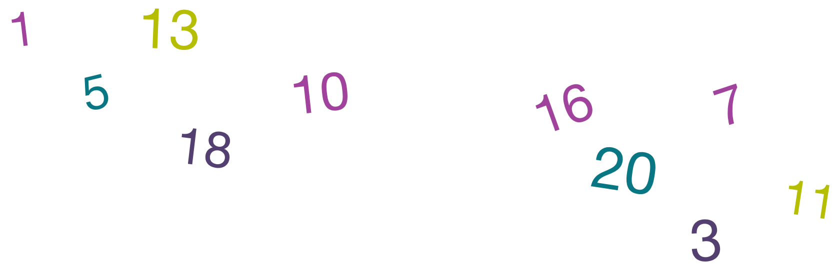
What might 'Stone Age' people count?

Try making up your own way of recording counting.



Take Three from Five

Choose any five positive whole numbers.



Will any set of five always include three numbers that will add up to a multiple of 3?

Temperature

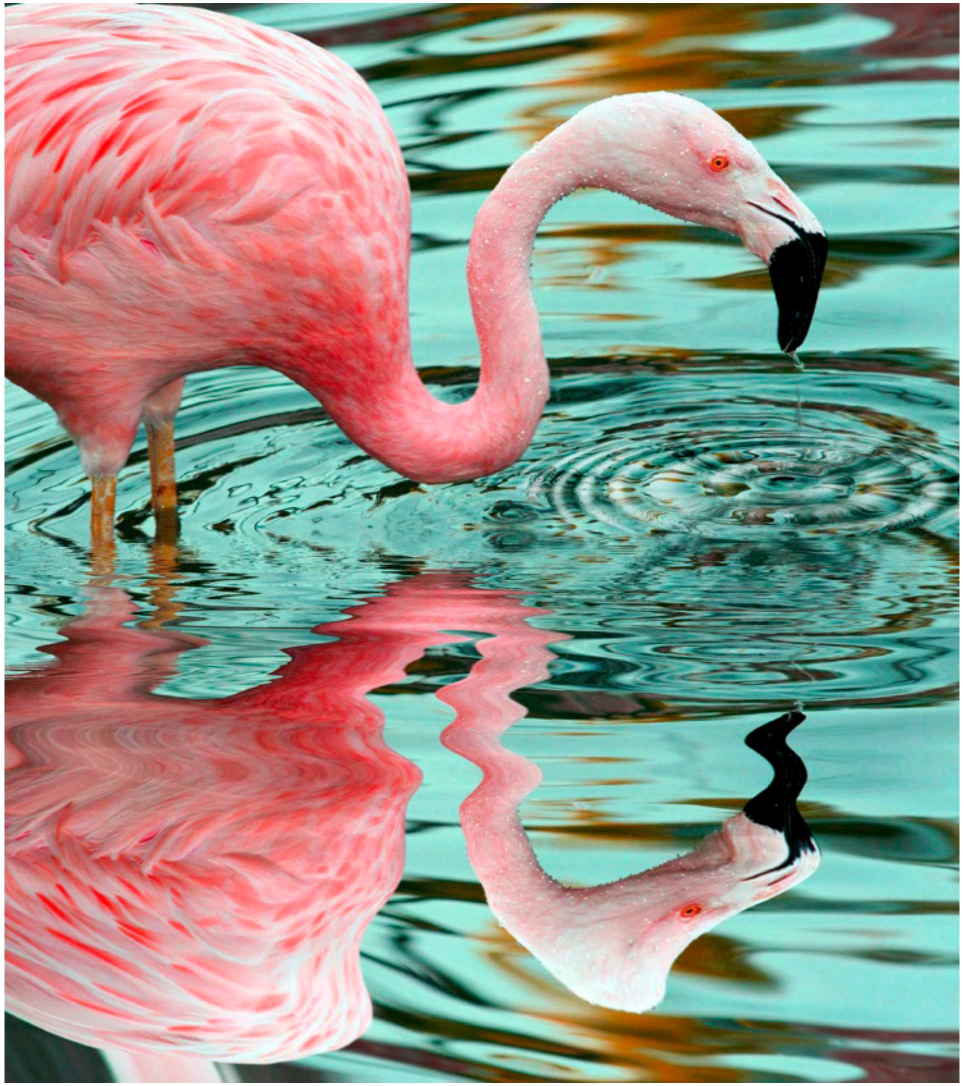
Temperature is often measured in Celsius (°C) or Fahrenheit (°F).

The freezing point of water is 0°C or 32°F.

The boiling point of water is 100°C or 212°F.

Is there a temperature at which the Celsius and Fahrenheit readings are the same?

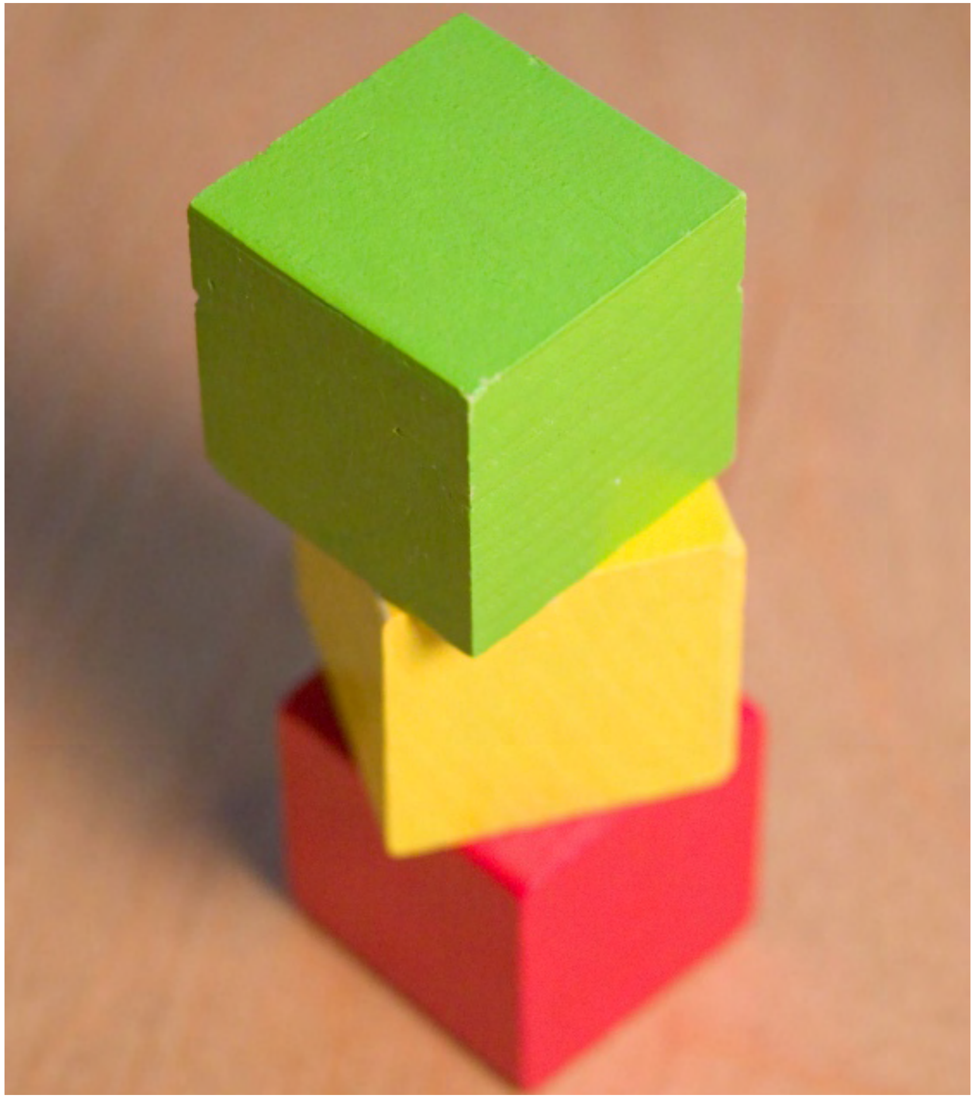
Think of Two Numbers



Think of two whole numbers under 10
Take one of them and add 1
Multiply by 5
Add 1 again
Double your answer
Subtract 1
Add your second number
Add 2
Double again
Subtract 8
Halve this number and tell me your answer

From your answer I can work out both your numbers very quickly. How?

Three Block Towers



How many different towers can you make using one red, one blue and one yellow block?

How many can you make if you have a green block as well?

Three Fruits

Quince, quonce and quance are three types of fruit.

7 quince weigh the same as 4 quance.

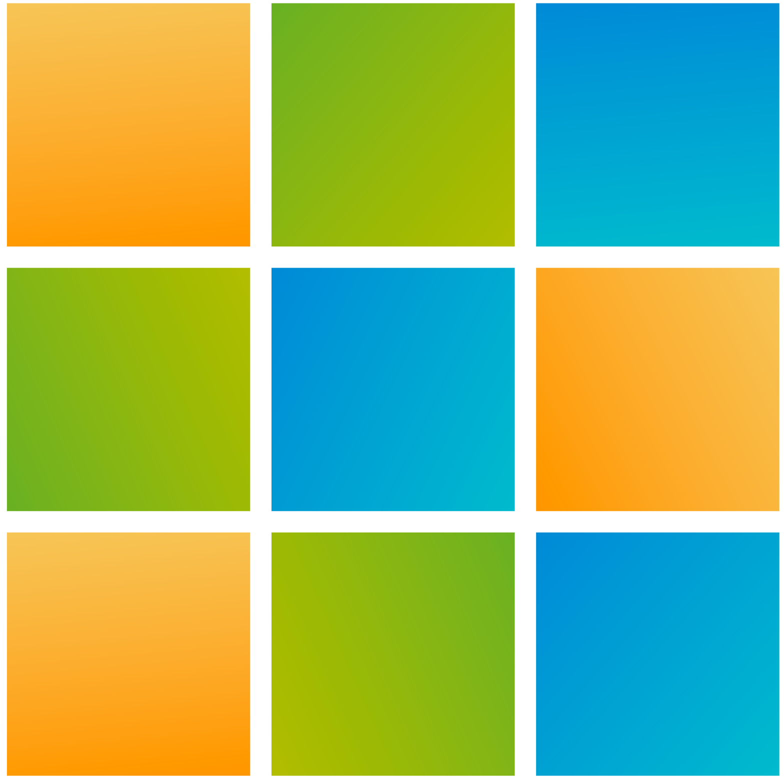
5 quance weigh the same as 6 quonce.

Put the fruit into order of weight, heaviest last.

Three Way Mix Up

Jack has three blue tiles, three yellow tiles and three green tiles.

He put them together in a square so that no two tiles of the same colour were beside each other.

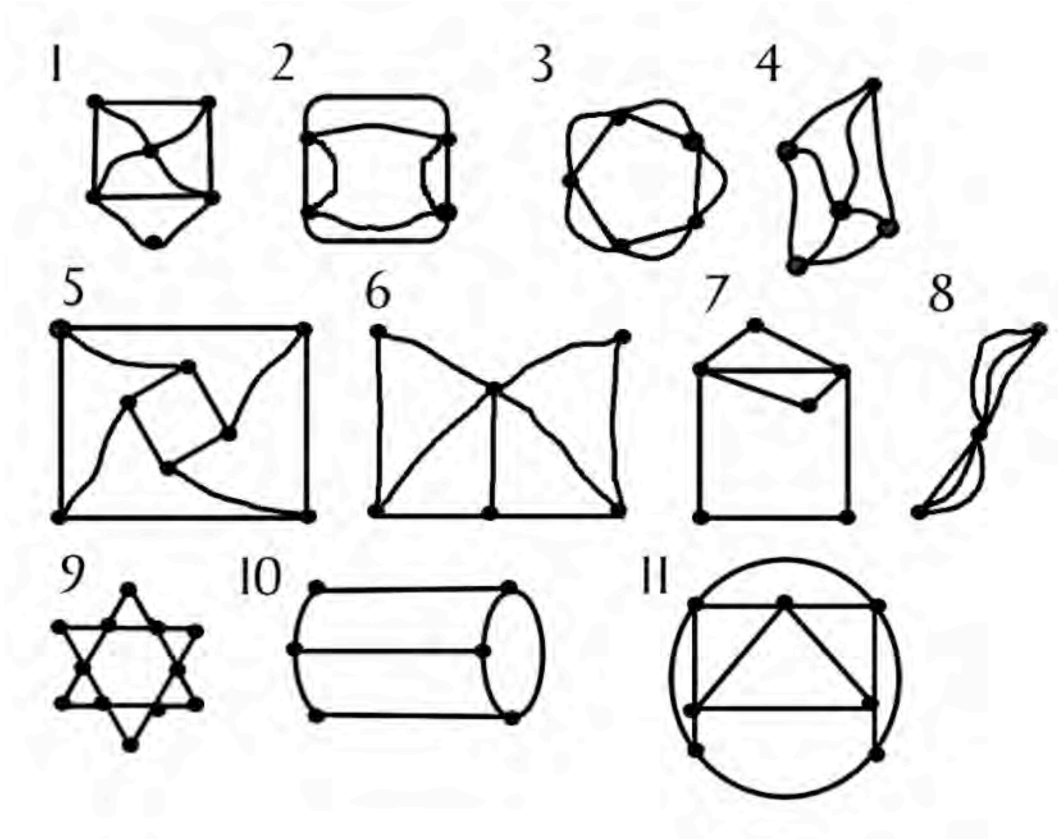


Can you find another way to do it?

Can you find ALL the ways to do it?

Tourism

Which of these diagrams can you copy without lifting your pen off the paper and without drawing any line twice?



Two and Two

How many solutions can you find to these two alphanumerics?

Each of the different letters stands for a different number.

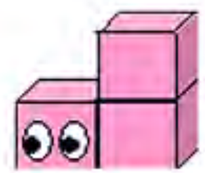
```
   ONE        TWO
 + ONE      + TWO
 -----      -----
   TWO       FOUR
 -----      -----
```

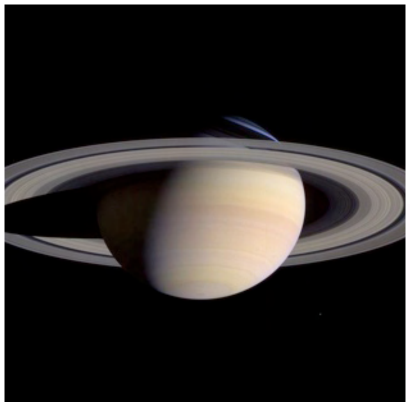
Wag Worms

When intergalactic Wag Worms are born they look just like a cube. 

Each year they grow another cube in any direction (except on their faces, of course). So a two-year-old Wag Worm might look like any of these:

So one shape a three-year-old Worm can be is this:





Can you find all the shapes a three-year-old Wag Worm could be?

Image of Saturn: NASA / Wikimedia Commons

Which is Biggest?

Which of these four expressions is largest?

$2n + 3$	$n + 10$
$\frac{1}{2}n + 1$	$3n - 14$

Is there a diagram you could draw that would help you decide?

(W)holy numbers

A church hymn book contains 700 hymns, numbered 1 to 700. Each Sunday the people in church sing four different hymns.

The numbers of the hymns are displayed in a frame using single digit boards. The board for 6 may be turned upside down to serve as a 9.

What is the minimum number of small boards that is needed to show any possible combination of four hymn numbers?

You Never Get a Six

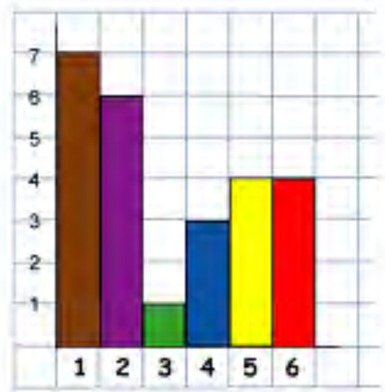
Tom, Vincent, Charlie and Edward were playing with dice.

They made lists of all their throws and then drew graphs of their results. They decided to make each of the numbers on the dice a different colour on the graphs.

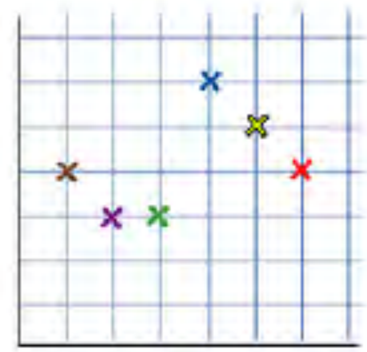
Who threw the most sixes?

How many of each number were thrown altogether?

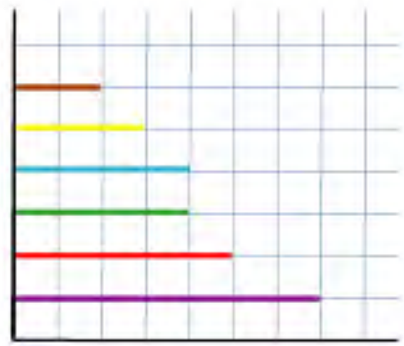
What percentage of the throws were sixes?



Edward's finished graph



Tom's unfinished graph



Charlie's unfinished graph

Vincent's unfinished pie chart