

The job of the teacher is to make it easy for students to learn. Or is it? **Alan Wigley** invites us to take a closer look at the curriculum we offer to learners of mathematics

MODELS FOR TEACHING MATHEMATICS

The current scene

One potential advantage of a National Curriculum is that, with the content at least partly specified and ordered, we can move our energies from consideration of the *what* to consideration of the *how*. It is a challenge for teachers to work together on effective ways of approaching chosen topics. How this is done is likely to have a more lasting effect on pupils' learning and their attitudes to the subject than the particular content selected. Perhaps, since of its very nature a national curriculum cannot be idiosyncratic and must compromise, it will seem rather conventional to forward-looking teachers. This does not make me pessimistic. With some teachers at least, I sense an emerging re-orientation in which a chosen published scheme, rather than defining the course to be followed, is being used more selectively to meet national curriculum requirements. This opens the possibility of teachers taking greater control over what they offer their students, potentially to the great gain of the latter.

In a previous article (MT132) I suggested three major issues on which we in ATM ought to be working during this decade:

- (i) resolving the content/process dichotomy;
- (ii) developing ways in which pupils can be helped to reflect on their learning;
- (iii) removing the unfortunate polarisation of the teacher's role into that of either instructor or facilitator.

These issues are germane to the whole educational debate and are not confined to mathematics. Hence the need to discuss them with fellow professionals and others with a public interest. More particularly, and despite changes over the years, I believe that we are far from achieving a consensus about approaches to teaching within the mathematics education community itself. Why, for example, do many authors of texts take on the impossible task of trying to create the whole

context for the learner on the written page when, as I believe, mathematics must necessarily be created 'in the air' of the classroom? I long to see more straightforward treatments of mathematical topics, enriched by descriptions of the historical and cultural context of the subject and with appropriate challenges for the reader. Let's have more dictionaries and reference books! Even the old-fashioned mixed bag of exercises is a good resource, particularly when pupils have to classify examples by type, sort out which they can solve and what methods are appropriate!

If we are to get our own house into better order, it is as important to tease out significant differences of interpretation as it is to emphasise similarities. In this article I shall first explore a model of teaching and learning which still seems to me to be too prevalent in mathematics classrooms. I shall consider some of the reasons why this model remains socially acceptable. I shall then describe an alternative model and indicate some of the ways in which it might be developed in the classroom.

The path-smoothing model

First, the main features of the model, the essential methodology of which is to smooth the path for the learner:

- 1 *The teacher or text states the kind of problem on which the class will be working.*

The teacher or text attempts to classify the subject matter into a limited number of categories and to present them one at a time. There is an implicit assumption that, from the exposition, pupils will recognise and identify with the nature of the problem being posed.

- 2 *Pupils are led through a method for tackling the problems.*

The key principle is to establish secure pathways for the pupils. Thus it is important to present ways of solving problems in a series of steps

which is as short as possible, and often only one approach is considered seriously. Teachers question pupils, but usually in order to lead them in a particular direction and to check that they are following.

- 3 *Pupils work on exercises to practise the methods given aimed at involving learners more actively.*

These are usually classified by the teacher or text writer and are graded for difficulty. Pupils repeat the taught processes until they can do so with the minimum of error.

- 4 *Revision*

Longer term failure is dealt with by returning to the same or similar subject matter throughout the course.

Although this model emphasises *repetitive* rather than *insightful* activities, almost all teachers who use it as their basic approach will also consciously offer some insightful experiences. They will, for example, attempt plausible explanations, or encourage pupils to gather data about particular cases before offering a generalisation. However, there is usually a pressure of time felt by teachers, and consequently by their pupils, to move on to the 'work', which is perceived as doing exercises. The teacher may find the time to offer explanations but not to provoke the debate needed to clarify meanings. Inevitably, pupils' perceptions remain unexamined if they passively agree to the arguments in order that work can proceed. So attempts to justify and explain, although genuine in intent, can fail to convey understanding to the pupils.

It is important to note that the model is perpetuated in most textbook schemes. Individualised schemes almost inevitably follow the model, because they are dependent on the pupil being able to take small manageable steps, without constantly referring to others. So does any approach which basically uses a sequence of pre-structured questions and does not give pupils the space to explore their own responses to situations or to participate in making significant choices for themselves. We even have structured investigations, which attempt to reduce exploratory work to a series of algorithms or pattern-spotting exercises!

Teachers who hold to this model of teaching and learning certainly exercise professional care for their pupils and help them to achieve an important measure of success in public examinations. This care is shown in a variety of ways: by providing a structured framework with a clear work pattern, by marking the pupil's work on a regular basis and explaining where the pupil has gone wrong, by being available to sort out difficulties as they arise. As to public examinations, these tend to fall into a set pattern over the years and are therefore often amenable to a path-smoothing approach. The model is also one which parents and the public can recognise – a popular, if only partial, view of how

learning takes place. These, perhaps, are the pre-eminent reasons as to why the model is so persistent in the face of an increasing body of knowledge and understanding about learning which tells a more complex story.

But limitations of the model may emerge in various ways. Sometimes learners flounder when presented with an unfamiliar problem, perhaps because they lack the strategies to explore the problem or the insight to recognise how it relates to problems which they have met before. From adults we hear comments such as 'I was no good at maths' or 'I could do the maths but never really understood what it was about'. On the one hand, we have a story of repeated failure and on the other, a lack of insight into mathematical relationships and their application to different contexts which has been only too prevalent in the adult population. Unfortunately being 'no good at maths' has itself become socially acceptable in some quarters, thus further perpetuating the model.

Some problems in teaching and learning mathematics

Before setting up an alternative model, I want briefly to consider some general issues.

There is a tendency in debate to polarise teaching and learning styles into one of two camps:

exploration	instruction
invented methods	given methods
creative	imitative
reasoned	rote
informal	formal
progressive	traditional
open	closed
process	content
talking (pupil)	talking (teacher)
listening (teacher)	listening (pupil)

The standard response is to declare oneself to favour a mixture of methods, neither entirely didactic nor entirely exploratory. But it is precisely here that the danger of a cosy consensus lies. The problem is not *whether* one should use a mix of methods (of that I have no doubt) but precisely *how* the blend should be achieved. Juxtaposing some open-ended tasks alongside a more prescriptive approach to much of the syllabus is not the kind of mix I have in mind! Neither is the path-smoothing model, even though it might sometimes blend the two approaches. When it comes to implementing an alternative model (what I shall call the challenging model), perhaps the chief demand on the teacher is to learn to live consciously with the creative tension which exists between exploration and instruction – in deciding, for example, whether to tell or not to tell about a particular matter.

A specific problem for teaching and learning

arises from the complex nature of mathematics itself. One aspect relates to the external 'meaning' of a mathematical idea, for example, what a 'half' of something is. It is important to be able to connect mathematics with aspects of the environment if we are to understand the subject and apply it to practical contexts. However, it is not as simple as that. Quite a lot of mathematics consists of invented systems with internally consistent sets of rules which cannot be abstracted from the world around us; the place value system for representing numbers is an example. Furthermore, to be competent in mathematics means partly to be able to operate fluently within the system, without constant reference to external meaning. For example, from an early age children build up experience of properties and relationships within the number system. Later, they learn how to manipulate algebraic statements, which are themselves generalisations of numerical relationships and thus a further step from 'reality'. Mathematical terms may have a multiplicity of meanings, with chains of interconnections. Thus a fraction can be interpreted as a part of a whole, as a ratio of two quantities, as one quantity divided by another; it has equivalent fraction, decimal and percentage forms.

Given this complexity it is small wonder that there has been an over-emphasis on rule-giving methods of teaching. How do you help students to develop their mathematics in meaningful ways? At what point is it appropriate to ignore other aspects in order to develop fluency within the system itself? And how do you do this without losing a sense of meaning and without placing an excessive burden on the memory? That is the challenge.

For many years there has been a feeling around that too many students are required to jump through hoops and are not able to think problems through for themselves. Particularly since the sixties, this has led to a search for better explanations or models for mathematical concepts which can be presented to pupils to help them understand. But this is not always effective. I will take just one example. If pupils fail to see the connection between the manipulations they make with the base 10 blocks and the marks they make on paper, then what useful purpose have the blocks served? Perhaps what pupils need here is less of a contrived explanation and more direct experience of the working system. This might be achieved, for example, with a computer program which merely cycles like a simple digital counter, so that students can infer the structure of the system by observing and discussing the patterns within the digits as they repeat. Some images, and not just the geometrical ones, are particularly powerful and pervasive – the cycling counter, the number line, the tabular array....Some of the work done in recent years, and written about in various issues of MT, suggests

that the role of the teacher might be to offer carefully selected experiences, in order to evoke images which can be developed and discussed. In this way the explaining, rule-creating or whatever, arises through sharing and discussing the experiences of learners; it is not externally imposed by the teacher.

Learners have to re-interpret what is offered and make sense of it for themselves. This is what it means to say that learning is an active process. It is not sufficient merely to work on problems and exercises. Learners reflect on experience, construct and test theories. Only then can they successfully integrate their knowledge and apply it in fresh situations. What we need then is a model for classroom practice which engages the learner by fostering a conjecturing atmosphere. (I assume that this is what attainment target one of the National Curriculum is trying to address.) Let us now attempt to outline such a model.

An alternative – the challenging model

The main features of the model are:

- 1 *The teacher presents a challenging context or problem and gives pupils time to work on it and make conjectures about methods or results. Often the teacher will have an aspect of the syllabus in mind, but this may not be declared to pupils at this stage.*

An important word here is challenge. The problem must be pitched at the right level, not too difficult, but more importantly, not too easy. The challenge may come from the complexity or the intriguing nature of the problem and the persistence needed to make progress with it, it may come from the variety of approaches which pupils bring to it, or from attempting to resolve the different perceptions which pupils have of a shared experience.

A second important word is time. It is crucial to give sufficient time for pupils to get into the problem – to recognise that it poses a challenge and that there may be a variety of approaches to it – so that discussion begins.

- 2 *Out of pupils' working is established a variety of ways which help to deal with the situation.*

Here the role of the teacher is again crucial – initially, in drawing out pupils' ideas, so that they can be shared within groups or by the class as a whole. At some stage the teacher may wish to offer some ideas of her own.

- 3 *Strategies which evolve are applied to a variety of problems – testing special cases, looking at related problems or extending the range of applications, developing some fluency in processes.*

Sometimes the syllabus requires the learning of more formal processes. The stimulus for this may be

a harder mathematical problem and may require exposition by the teacher. However, the pupil will have the context of previous work to which more advanced techniques can be related.

- 4 *A variety of techniques is used to help pupils to review their work, and to identify more clearly what they have learned, how it connects together and how it relates to other knowledge.*

Longer term failure is dealt with by ensuring that any return to the same subject matter encourages a different point of view and does not just go over the same ground in the same way. The model places a strong emphasis on the learner gaining new insights, and the time required for reflection is considered to be fully justified.

Thus, at the heart of this model is the challenge to the learner. The teacher's function is not to remove all difficulties but to present the initial challenge and then to support the class in working on it. This may be a stumbling block if, as is often the case, pupils have different expectations of teachers and do not feel that they are being appropriately cared for. There seem to me to be three essential elements in developing the necessary supportive framework: to encourage collaborative work, to set more open tasks relating to global or key aspects of the syllabus, and to provide ways in which pupils can reflect on what they have done and relate aspects together. To achieve a necessary measure of teacher involvement, it is often preferable that the whole class should be working within the same topic. However, this does not preclude groups of pupils selecting their own topic of study from time to time, particularly once an effective way of working has been established.

For all but the youngest pupils, where paired work is perhaps more feasible, it is possible to arrange for small groups of say four students to work collaboratively together. Each group may work on their own strategies to solve a common problem, on different aspects of a common topic or, sometimes, on different topics. Within this context the teacher can work in depth with particular groups, bring groups together to share ideas, and inject new ideas when required. Individual work will still occur, both as a contribution to the group effort and to meet individual needs. However, the greatest challenge to the teacher is to encourage good discussion within the groups and develop a climate in which pupils can see each other as a first resort for help and support.

It helps if chosen topics are based on a major area of the syllabus, focus on central ideas, and extend over a lengthy period of time – perhaps even up to half a term. A substantial task can thus be presented, in which pupils help to: define the problem; develop ways of tackling it; generate examples to test a theory or practise a method; predict and make generalisations; and explore

further applications. Moving from exploratory work into new areas of knowledge requires specific strategies, such as sharing ideas about a carefully posed and challenging problem. Direct input by the teacher will sometimes be appropriate. In this way of working, a few key starting points are needed, sometimes supported with written sheets. A wide range of texts and reference books can be dipped into as required.

Another essential feature is reviewing, which is the term I use to describe various reflective activities, in which pupils step back from the immediate situation, and consider what they have done and how it relates to other aspects of their learning. Reflective activities can take the form of talking and writing about the processes pupils have gone through, making posters and reporting to the class, drawing up concept maps of a topic, and sharing attainment targets. Reviewing motivates and informs future learning activities and fosters general study skills.

Concluding remarks

I want to stress that the challenging model can be applied to the conventional syllabus and examinations in conventional (well, fairly conventional!) classrooms. It does not require extravagant resources, pupils of special aptitude or teachers of rare ability. It cannot be prescribed by the government, nor can it be proscribed by them! The benefits can be better learning and more positive attitudes towards the subject. Having described the key features of the model, it needs to be developed and illustrated with a variety of examples. I would be pleased if readers of MT responded to this article by giving examples to illustrate the model in practice in particular classrooms. There is the need to develop both the strategy of the challenging model and the tactics which might be employed with specific mathematical topics. Should we be seeking to establish this approach more widely in classrooms by the end of the century?

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