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Niharika Paul Phase Space Age 9.

For the function $f(x)$ ~~ ~~ $f'(x) = \lim_{z \to 0} \frac{f(x+z) - f(x)}{z}$

I define a symbol ~~ ~~ were αx = change x ~~betwe~~ were x goes from x_1 to x and change $\to 0$.

$\therefore$ I can define $f'(x)$ as $\frac{\alpha f}{\alpha x}$

Using this idea I have produced a slope table:

~~Slope~~ Function	Slope
$f(x) = A$	0
$f(x) = x$	1
$f(x) = Ax + B$	A
$f(x) = x^A$	Ax^{A-1} (I can prove this using induction)
$f(x) = e^x$	e^x (I can prove this using the series form)
$f(x) = \sin x$	$\cos x$
$f(x) = \cos x$	$-\sin x$
$f(x) = \log x$	x^{-1} (I have taken this from other slope tables)

In the general case:

$v = f(x)$ here v is velocity and x is displacement

$\therefore \frac{\alpha x}{\alpha t} = v$ here t is time

or $\frac{\alpha x}{\alpha t} = f(x)$

or $\frac{\alpha x}{f(x)} = \alpha t$

However I do not want to work with αs because they are small parts.

So let $\frac{\alpha x}{f(x)} = \alpha z$

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$\therefore \; \alpha z = \alpha t$

Now, the bounds of t are $[t_1, t]$

$\therefore \; Z(t) - Z(t_1) = t - t_1$ —— (I)

Now, we need to relate Z to x

We know that $\alpha z = \dfrac{\alpha x}{f(x)}$

or $\dfrac{\alpha z}{\alpha x} = \dfrac{1}{f(x)}$

So we must find a function with the slope of $\dfrac{1}{f(x)}$.

Let us call this function $g(x)$

From (I) we find that:

$[g(x)](t) - [g(x)](t_1) = t - t_1$

This gives us a relationship between x and t.

1. In this function $v = k$. Here k is a constant, $k \in \mathbb{R}$

Using the terms in the general case we find:

$\dfrac{\alpha z}{\alpha x} = \dfrac{1}{k}$

$\therefore \; z = \dfrac{x}{k} + c$ here c is a constant $c \in \mathbb{R}$.

$\therefore \; t - t_1 = \dfrac{x}{k} + c\,(t) - \dfrac{x}{k} + c\,(t_1)$

2. In this plot $v = kx$.

$\therefore \; \dfrac{\alpha z}{\alpha x} = \dfrac{1}{kx}$

$\therefore \; z = \cancel{\frac{1}{k}x^2}\; \log kx$

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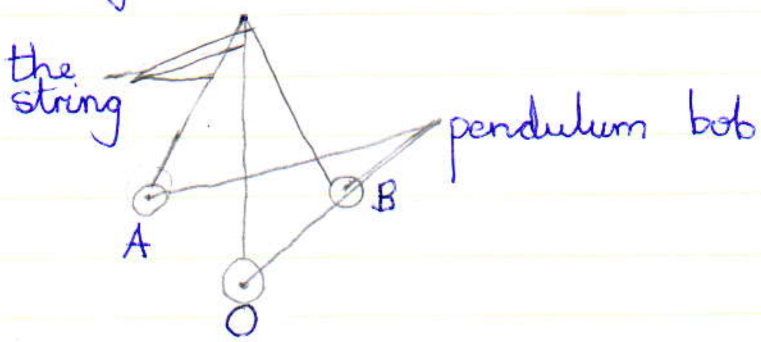
or $\cancel{t - t_1 = 1/R_x^2(t) \; 1/R_x^2(t_1)}$ $t - t_1 = \log[R_x(t)] - \log[R_x(t_1)]$

3. I was doing Stem Nrich and found out that the elliptic function was the function ~~between~~ of the velocity of the pendulum (v) against its displacement (x). Here is why:

First consider this diagram:



First ~~When~~ the bob is at O $v = v_{max}$ and $x = 0$. Then the pendulum goes to A, there ~~the~~ $v = 0$ and $x = x_{max}$. Then at O $v = -v_{max}$ and $x = 0$. Then the pendulum goes to B where $v = 0$ and $x = -x_{max}$. This carries on in a cycle and forms an ellipse.

4. This function is one which I do not know about.

5. This ~~rhomat~~ rhombic function consists of 4 parts. In each part $v = Rx + c$, here R, c are constants, $R, c \in \mathbb{R}$

$$\therefore \frac{dz}{dx} = \frac{1}{Rx+c}$$

$$\therefore z = \frac{\log[Rx + c]}{R}$$

$$\therefore t - t_1 = \frac{\log[Rx(t) + c]}{R} - \frac{\log[Rx(t_1) + c]}{R}$$