The Tower of Hanoi puzzle consists of three pegs and a stack of discs on the first peg, arranged in order of size. The object is to move all of the discs to another peg. However, only one disc can be moved at a time, and a disc cannot be placed on top of a smaller disc.

Here are Alison's first 4 moves when she has 3 discs:

1) What is the smallest number of moves needed to solve the Tower of Hanoi with one disc? Two discs? Three discs? ...

   What do you notice about the way the number of moves increases?
   Can you explain any patterns you find?

2) With two discs, the Tower of Hanoi requires 3 moves. Can you use this to work out how many moves would be needed with three discs?

   With four discs, the Tower of Hanoi requires 15 moves. Can you use this to work out how many moves would be needed with five discs?

   In general, can you describe a way of working out how many moves are needed when one extra disc is added?

3) Explain how you could work out the number of moves needed for the Tower of Hanoi puzzle with \( n \) discs.

**Extension:**

There is a legend that a 64-disc version of the Tower of Hanoi is being played out in a temple, and when the final move is made, the world will come to an end. If one move is made each second, how long would it take to solve?

If the first disc was moved at the very beginning of time, should we worry yet?