http://nrich.maths.org/public/viewer.php?obj_id=6089

NRICH

Integrating Rich Tasks

To find the materials go to the website:

http://nrich.maths.org

On the top right-hand side click on “Courses”.

Then click on the link to the Introduction to Integrating Rich Tasks.

Draft materials, 11/11/08

Integrating Rich Tasks

Introduction

This series of professional development resources is designed to support teachers working together, thinking about, and integrating rich tasks into classroom practice.

The resources are divided into four phases of development giving time for reflection and practice. They have been designed to be tackled in order but we are aware that colleagues will be starting from different places and may wish to step into and out of the activities according to their particular need.

Many of the resources involve using various materials. These documents are found in the appendices.

Phase 1 - Thinking about rich tasks, problem-solving and higher-order thinking skills

Activity 1.1	What makes a task rich? In this activity you will try out some problems and then identify what makes them "rich".
Activity 1.2	How can we encourage higher-order thinking skills?
Activity 1.3	What is meant by higher-order thinking skills (HOTS)?
Activity 1.4	How do higher-order thinking skills relate to rich tasks and problem solving?
Activity 1.5	How do pupils progress in their problem solving?

Phase 2 - Using rich tasks in the classroom

Activity 2.1	What do teachers do to support learners engaging with rich tasks?
Activity 2.2	'HOTting up' your existing classroom materials.

Phase 3 - Integrating rich tasks into the whole curriculum

Activity 3	Integrating rich tasks into the whole curriculum.

Phase 4 - Reflection and review

Activity 4.1	Peer observation
Activity 4.2	Evaluating a theme
Activity 4.3	Thinking about what to do next

Activity 1.1 - What makes a task rich?

To help to answer this question we suggest that you try some unfamiliar problems yourself. In this activity you are first asked to spend some time working on a problem, ideally with a colleague, before trying to identify what we mean by a 'rich' task and what would make doing the particular problem you have studied a 'rich' activity for your pupils.

You will need the following resources:

- List of attributes of rich tasks [*Appendix 1*]
- Blank 'rich task' template [*Appendix 2*]
- Suggested NRICH problem aimed at KS1 – Eggs in Baskets [*Appendix 3*]
- Exemplar template for Eggs in Baskets [*Appendix 4*]
- Suggested NRICH problem aimed at KS2 – Got It [*Appendix 5*]
- Exemplar template for Got It [*Appendix 6*]

What to do:

- Try one of the suggested problems on your own or with another colleague. [*Appendix 3* or *Appendix 5*]
- Look at the short list of attributes of a rich task described in *Appendix 1*. Discuss how they link to your own experiences when solving the problem.
- Use the blank template (*Appendix 2*), which lists the attributes of a rich task, to make your own notes about why the problem you have worked on could be described as a rich task. Remember that a rich task does not have to have all the attributes and much will depend on how it is used in the classroom.
- Join with other colleagues and compare your template with theirs.
- You might like to finish by looking at the *completed template* for the problem you tried (*Appendix 4* or *Appendix 6*). These represent our own experiences of using the tasks in classrooms so they may look different to your own. There are of course many answers. It would also be worth looking at the notes section of the problem on the website.

Activity 1.2 – How can we encourage higher-order thinking skills?

To help to answer this question here are two tasks for you to do which we hope will help you to:

- distinguish between problems that encourage higher-order thinking skills and problems which don't
- develop problems of your own that support higher-order thinking skills

In this activity we shall focus on what we are looking for in our pupils when they are engaged in using higher-order thinking skills (HOTS).

You will need the following resources:

- Set of cards for matching [*Appendix 7*]
- Document of strategies for modifying tasks [*Appendix 8*]

The first task

- The cards in *Appendix 7* contain some lower-order questions and, focusing on the same mathematical topic, some more challenging questions - ones that require higher-order thinking skills. Pair them up.
- Now, with colleagues, answer the following questions:
  - What do you think higher-order thinking skills are?
  - What do tasks that encourage higher-order thinking skills look like?
- Look at these notes on higher-order thinking skills and compare them with your ideas. Are there any major differences? What is your response to those differences?

The second task

- Instead of replacing a lower-order problem with a different problem, we can often modify it. How can we adapt lower-order maths problems so they promote HOTS? *Appendix 8* outlines four key strategies that will help to increase the challenge of standard questions in the classroom:
  - Here's the answer, what could the question be?
  - Make up your own ...
  - What if ...?
  - All answers
- Look at a problem you have recently set one of your classes and discuss how it could be transformed into one requiring higher-order thinking skills. Jot down your ideas and keep them for Activity 2.2

Activity 1.3 – What is meant by higher-order thinking skills (HOTS)?

This task, and the one following it, builds on Activity 1.2

You will need the following resource:

- Bloom taxonomy descriptor cards [*Appendix 9*]

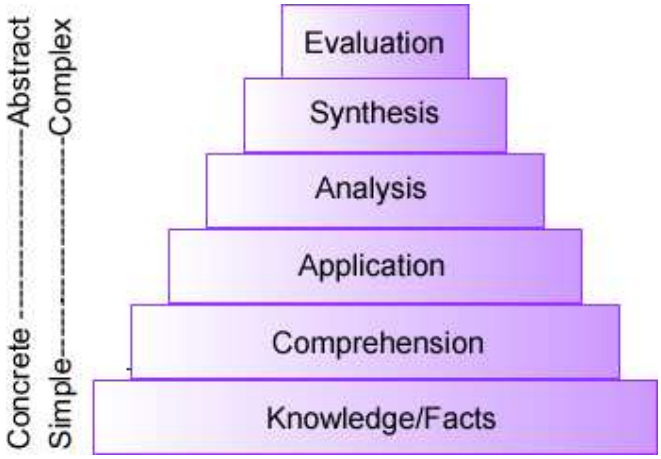
Task

- Bloom's taxonomy is a hierarchy of skills that reflects growing complexity and ability to use higher-order thinking skills. The descriptions of the skills are listed in *Appendix 9*]. Try to put them in order of complexity. When you have done this, and discussed what you think are the most challenging activities, you might wish to look at the pyramid of skills known as Bloom's taxonomy at the foot of this page.

- Think of a lesson you have recently given - what level of thinking were you expecting of your pupils?

'Bloom's Taxonomy'

Bloom's Taxonomy is a hierarchy of skills that reflects growing complexity and ability to use higher-order thinking skills (HOTS).



Adapted from: Bloom, B.S. (Ed.) (1956) Taxonomy of educational objectives: The classification of educational goals: Handbook I, cognitive domain. New York ; Toronto: Longmans, Green.

Activity 1.4 – How do higher-order thinking skills (HOTS) relate to rich tasks and problem solving?

This task aims to identify how rich tasks and problem solving fit together.

You will need the following resources:

- Bloom taxonomy descriptor cards [*Appendix 9*]
- Problem-solving cycle cards [*Appendix 10*]
- Problem-solving cycle [*Appendix 11*]
- Rich task cards [*Appendix 12*]

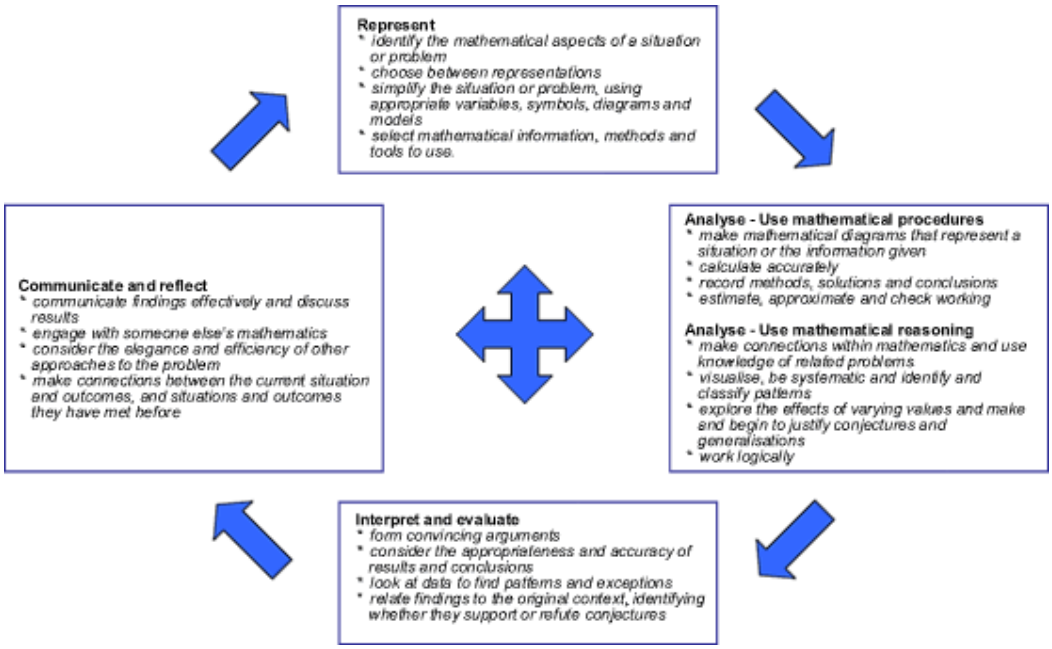
Higher-order thinking skills are not about mathematical content knowledge. Just like it is possible to engage in very hard questions that involve a high level of content knowledge but few problem-solving skills, it is also possible to identify very difficult problems that only need very low levels of mathematical content knowledge. In the former case, you are going to need well-tuned knowledge skills and in the latter, your HOTS.

What is a problem?

A problem is something you do not immediately know how to solve. There is a gap between where you are and even getting started on a path to a solution. This means that something that is a problem to your students is something that they cannot get to grips with immediately and requires thinking and playing time. By playing with the mathematics, patterns and connections often reveal themselves. We need to arm our pupils with a repertoire of skills to help them step into problems independently rather than immediately turning to us as teachers to ask what to do! We can begin by selecting problems with engaging starting points which invite pupils to step in (such as a game). Once they get started, the richness comes from what happens next. Ideas begin to emerge from playing with the initial situation and sometimes from posing problems of their own.

What is problem solving?

The need to apply problem-solving techniques to a problem is an indicator that it has the potential to be a rich task. Problem solving requires you to have a problem to solve, which may be one you have been given or one you have posed for yourself. The activity that we call 'problem solving' is a complex one and can be considered as a cycle of activity (though the cycle often requires us to move backward and forward whilst maintaining a general sense of direction). There are many models of the problem solving cycle. Possibly the most well known is the one described by Polya in his book *How to Solve It* (1957), which is a must-read for those of us interested in improving our pupils' problem-solving skills. Here is one we use at NRICH:



(Appendix 11 is a larger version of the above which might be easier to read.)

The application of the problem-solving cycle is a high-order skill. Evidence suggests that few pupils utilise the problem-solving cycle effectively. One important thing to note is the emphasis the cycle places on the high-order thinking skills described by Bloom. It is therefore not surprising that most pupils do not naturally have a sense of where they are and what they might do next. One of our aims when teaching mathematics is to help pupils become familiar with this process and have confidence to use it.
See Polya, G. (1957). How to Solve it, Princeton University Press.

How do rich tasks, the problem-solving cycle and higher-order thinking skills fit together?

Here is a task to help answer this question.

Task

- Cut out the problem-solving cycle cards (*Appendix 10*) and lay them out.
- Link them with the rich task description cards (*Appendix 12*) and with the different aspects of Bloom's taxonomy (*Appendix 9*).

We feel that any problem has the potential to be a rich task but this depends on us as teachers offering those opportunities to our pupils. We will talk about this in Activity 2.1 .

Activity 1.5 – How do pupils progress in their problem solving?

In the previous activity you were asked to think about the connections between higher-order thinking skills, problem solving and rich tasks. In the next set of activities we want to think about how we can support our pupils in problem solving.

You will need the following resources:

- Progression cards [*Appendix 13*]
- Problem-solving cycle cards [*Appendix 10*]
- For reference you may want to refer to the progression list [*Appendix 14*]

Task

We have based this activity on the National Strategy's Primary Framework Assessment Guidelines. We are not asking you to think about assessment but about process skills and progression. The guidelines are based on three areas: problem solving, reasoning and communicating.

There are two parts to this task. There is no 'right answer' to either part but the activities are designed to make you think about:

- the mathematical thinking and problem-solving skills you want your learners to develop
- the sorts of things your pupils will be doing
- the development of thinking and problem-solving skills over time (progression)

It is the discussion you have as you undertake the task which is key. By making sense of phrases and describing what you mean by them in your own words you will come to your own view about how they inform what you are trying to help your pupils to learn.

First you will need a set of the problem-solving cycle cards (*Appendix 10*) and of the progression cards (*Appendix 13*).
[The Progression Cards are based on lists for Levels 2, 3, 4 and 5 so you might like to think about what would come before L2 and after L5.]

Lay the cycle cards out and then distribute the progression cards amongst them. There will be quite a lot of discussion about what some of these mean. Remember that there is no right answer and a lot depends on your interpretation of a card's meaning. In the end you should put each card under the heading that feels like the 'best fit'. Do not agonise for too long on each card - you can change your mind at any time. *When we did this task at NRICH we moved things around quite a lot during the second part of the task!*

The **second** part of the task is about ordering the cards under each of the five process headings. The aim of this part of the task is for you to think about progression. What would you expect learners at different stages to be able to do? *When we did this task we found it useful to group cards that seemed to be about similar things together before trying to order them. So, for example, under Analysis-Reasoning we found a few cards that seemed to be about 'organising' so we pulled these out and put them in order* .

The lists are not meant to be exhaustive so you might want to add some cards of your own.

When you have finished the tasks you might find it useful to refer to the progression list (*Appendix 14*) as this will enable you to map what you have done to the Strategy document.

Activity 2.1 – What do teachers do to support learners engaging with rich tasks?

This pair of tasks build particularly on Activities 1.1, 1.3 & 1.4. The aim is to look at a problem and think about what we can do to help make it rich. This is because, regardless of a problem's potential, the way it is used affects its richness.

You will need the following resources:

- Rich task cards [*Appendix 12*]
- Rich task sheet - what teachers can do [*Appendix 15*]
- What teachers do - master template sheet [*Appendix 16*]
- Suggested NRICH problem [*Appendix 18*]
- What teachers do - Magic Vs sheet [*Appendix 17*]

Task 1

Stick each of the rich task cards (*Appendix 12*) on a separate A3 sheet. As a group, move around the sheets and add ideas for what you could do as teachers to help support each aspect of a rich task. This will be very general at this stage. If you need help some ideas are given on the what teachers can do sheet (*Appendix 15*). These ideas will become more specific when applied to a particular problem.

Task 2

Work on the NRICH problem Magic Vs (*Appendix 18*) so you feel confident that you know it well.

- Fill in the column of the master sheet (*Appendix 16*) labelled 'What pupils could do'.
- Now fill in the column 'What teachers might do'. As you do this, think about the sorts of things you might do in the lesson to encourage pupils to tackle the problem and behave in the ways you have suggested in the middle column.

Appendix 17 is what we produced when we tried this at NRICH.

Activity 2.2 – 'HOTting up' your existing classroom materials

So far we have given you activities to work with that are on the NRICH website. However, you probably have many activities you use in your own lessons that have the potential to be rich, or richer. The aim of this activity is to draw your attention to those problems and think about how you can use them to develop higher-order thinking skills and problem-solving skills, and what you might do to support this in the classroom.

You will need the following resources:

- What teachers do - master template sheet [*Appendix 16*]
- Notes you made during the second task in Activity 1.2

What to do:

This links to the work on higher-order thinking skills and Activity 2.1 on rich tasks. Here the aim is for us to think about what we can do as teachers with problems we already use. The emphasis is on what we do in the classroom rather than adjusting the problem itself as we did in Activity 1.2.

Retrieve your jottings from the second task in Activity 1.2 and then, working in a pair, consider what you would do to as a teacher to support this problem. Use the blank template (*Appendix 16*) and the ideas of Activity 2.1 (where we did a similar task for Magic V's) to help.

Why not share any good ideas with us at NRICH by emailing us?
nrich@damtp.cam.ac.uk

Activity 3 – Integrating rich tasks into the whole curriculum

The aim of this activity is to integrate some rich tasks into curriculum planning. Although there are other possibilities, at this stage we will look at two sources for these tasks:

- NRICH
- exisiting schemes of work

All the work we have done so far should feed into this activity, which is designed to be the starting point for a longer period of planning and development. The long-term aim is for you to think about your teaching and how it can be enhanced, but to start with you will need to select something that is realistic and achievable. You can always extend what you do at a later date.

You will need the following resources:

- Your existing scheme of work
- The NRICH curriculum mapping documents [*Appendix 19* for KS1 and *Appendix 20* for KS2 but the versions online will be more recent]
- The NRICH site - particularly the Maths finder, which you can find at http://nrich.maths.org/public/leg.php

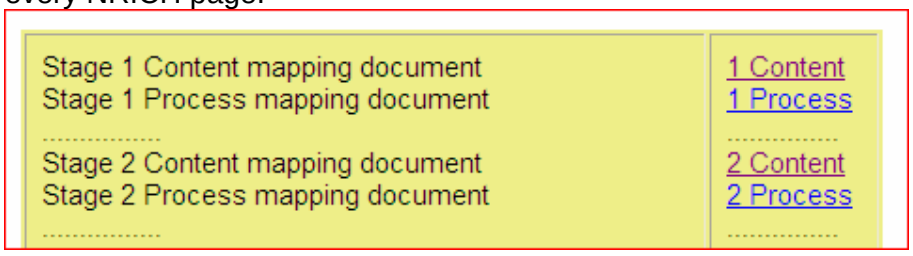
Task 1

First a reminder that we are not assuming that you are going to change everything now, you are just making a start. For this reason, we suggest you could begin by planning for a mathematical topic that you will teach this term.

There are many different approaches to planning for the integration of rich tasks, for example you could:

- Look at your current scheme of work and use the content mapping documents to find problems that are a good fit with the particular topic you are covering.
- Consider what using and applying skills you want your pupils to develop and use the process mapping documents to identify appropriate problems. You might use these as one-off problems but they will also address subject content knowledge so why not use them when you are covering that topic in your scheme of work?
- Identify a theme to work on for a longer period of time. Examples of themes are:
    - problems that employ several aspects of content knowledge (e.g. factors and multiples)
    - the development of problem-solving skills (the whole process)
    - the development of particular mathematical thinking skills (e.g. 'working systematically' or 'visualising')
    - an application of mathematics (e.g. time and its measurement)

The mapping documents will help with the first two approaches suggested above (content and process blocks). There are no specific documents designed to support the third approach but the Maths finder and Packages on NRICH can help. There is also a 'search NRICH' option found at the top of every NRICH page.



You may want to access the Curriculum mapping documents on the website, or use the versions provided in *Appendices 19 &* 20 (note the versions online will be the most up-to-date).

Alternatively (or in addition) you could identify potentially rich tasks you are already using and extend them in the ways you did in Activities 1.1 and 1.2.

What next? - Task 2

Whichever approach you take, for each problem you will need to spend time thinking about why it is rich (for the problems from the NRICH mapping documents this has already been done) and what you will need to do in the classroom to support pupils in making the most of them (as in Activity 2.1). As you try things out, you will refine ideas and will feed back to your colleagues what worked well and why.
This is no small task and that it is why it is worth starting with something small and achievable rather than trying to do everything all at once.

We will look at evaluation in the next Activity.

Activity 4.1 – Assessing your embedding of rich tasks into the curriculum: Peer observation

Having started to embed rich tasks into your scheme of work you will need to:

- assess what works and what does not
- make decisions on how to extend your mapping
- consider what further support you might need

This Activity, along with Activities 4.2 and 4.3 are designed to help you with the above.

The best way to go about evaluating and reviewing a particular lesson is to work with a colleague. However, what is suggested here can be used as a means of self-reflection. Before the lesson you will need to prepare:

- Your lesson plan
- The "what teachers do" sheet [*Appendix 16*]

Either use this opportunity to do some peer observation, with a colleague using the prepared observation/reflection sheet, or during and after the lesson use the sheet to jot down some notes of your own.

Discuss or reflect on:

- what was successful
- what you would do differently next time
- what key things pupils did that could be highlighted or drawn out more in future

Use this to inform planning for next time.

Activity 4.2 – Assessing your embedding of rich tasks into the curriculum: Evaluating a theme

Undertaking the same sort of observation/reflection activity as you did in Activity 4.1 for individual lessons or a group of lessons is an important part of evaluating the success of a theme or series of linked lessons within a gven topic. In addition however, you need to look at the group of lessons more holistically.

To do this you will need to consider:

- the aims or learning objectives for this group of lessons including:
  - what content you were hoping to cover
  - what using and applying/problem-solving skills you were hoping to address
  - what connections you were hoping to make
  - which aims were met/not met. Try to describe why and, where appropriate, how things might be improved. This might involve being more realistic about your aims or thinking of other ways in which you might approach the theme or support pupils whilst working on a theme.

- how the pupils responded over all:
  - did they enjoy it?
  - did they reach the level of working you expected?
  - did the work cater for their individual needs (were the support and extension ideas and materials appropriate)?

Then update your planning documents:

- How you will modify the theme in future? This might involve removing it from your scheme of work or revising the 'what teachers do' sheet and lesson plan.
- List your recommended next steps. Include key points for colleagues who might try the same theme themselves.

Activity 4.3 – Assessing your embedding of rich tasks into the curriculum: Thinking about what to do next

Build on your experiences, adding in new material and trying it out. Using the mapping documents can ensure a range of experiences for your pupils. It is not enough to employ a concept or process once, you will need to revisit these again and again, each time thinking about how the pupils will develop. For example, when considering problem-solving skills, pupils will develop in different ways, such as:

- Becoming more independent with you having to do less supporting in order for them to think of ideas of their own.
- Applying more sophisticated content knowledge
- Being more equipped to talk about their mathematics
- More able to apply what they know in less familiar settings
- Better able to make connections with things they have done before
- Showing greater sophistication and organisation in their recording methods.

How does your scheme of work allow this to happen?

The important thing to do is:

Reflect - evaluate - modify if necessary

At NRICH we are really interested in finding our more about your experiences. Do email us so we can share your ideas and findings with others.
nrich@damtp.cam.ac.uk

Appendix 1	for Activity 1.1	a copy for each pair of teachers

RICH TASKS

Current research evidence indicates that students who are given opportunities to work on their problem solving enjoy the subject more, are more confident and are more likely to continue studying mathematics, or mathematics related subjects, beyond 16. Most importantly, there is also evidence that they do better in standard tests.

Rich tasks can enable pupils to:

- step into them even when the route to a solution is unclear, getting started and exploring is made accessible to pupils of wide ranging abilities
- pose as well as solve problems, make conjectures
- work at a range of levels
- extend knowledge or apply knowledge in new contexts
- allow for different methods
- have opportunities to broaden their problem-solving skills
- deepen and broaden mathematical content knowledge
- have potential to reveal underlying principles or make connections between areas of mathematics
- include intriguing contexts
- have opportunities to observe other people being mathematical or see the role of mathematics within cultural settings

See also: http://nrich.maths.org/public/viewer.php?obj_id=5662

Appendix 2	for Activity 1.1	a copy for each teacher

Rich task template: Why is …… a rich task?

Step into a problem even when the route to a solution is unclear (see definition of "a problem" below), getting started and exploring is made accessible to pupils of wide ranging abilities.	
Pose as well as solve problems, make conjectures	
Work at a range of levels	
Extend knowledge or apply knowledge in new contexts	
Allow for different methods	
Offer opportunities to broaden students' problem-solving skills	
Deepen and broaden mathematical content knowledge	
Have potential to reveal underlying principles or make connections between areas of mathematics	
Include intriguing contexts	
Offer opportunities to observe other people being mathematical or the role of mathematics within cultural settings	

Appendix 3	for Activity 1.1	a copy for each teacher of either appendices 3&4, or of appendices 5&6

This is a classroom task from the NRICH website.
Notes about the task, including downloadable SMARTboard notebooks and other materials are available on the site from: tinyurl.com/6ysepn

To download the SMARTboard file, click on the link. Then select "Save As…". It may be necessary to type ".notebook" at the end of the filename and to change the filetype to "All files". Please check that you are running a new enough version of the SMARTboard software (version 9.5 or later).

'Eggs in Baskets'

There are three baskets, a brown one, a red one and a pink one, holding a total of ten eggs.

The Brown basket has one more egg in it than the Red basket.

The Red basket has three eggs less than the Pink basket.

How many eggs are in each basket?

Appendix 4	for Activity 1.1	a copy for each teacher of either appendices 3&4, or of appendices 5&6

Exemplar template for Eggs in Baskets, showing how it can be used in the classroom.

The version online include links to view the video-clips and images.

Aspects of a rich task	Ideas for teacher support
Introducing the task so that the children can get started	The teacher encourages some general exploration of the situation by changing the context (sweets rather than eggs) and simplifying the problem (only 6 in total, as opposed to 10). Rather than having three unknowns to begin with, as in the NRICH problem, children start with one unknown (how many sweets in the third bag?) and then two (what could be in the second and third bags?).
The problem allows children to make conjectures	Clip "Eggs1.wmv" When shown the bag of four sweets, the children immediately begin to make conjectures. One suggests the other two bags will have one sweet each "because 2 and 4 make 6". Then another pupil suggests that there could be 2 in the second bag and zero in the other.
The task allows children to work at a range of levels	Clip "Eggs2.wmv" Here the resources provided allow this child to work on the problem in the way he feels comfortable, which is a good assessment opportunity for the teacher. Image "EggsA.gif" This learner has recorded the possible combinations using number sentences and has worked in a very systematic way. Note the sum which has been squeezed in near the top of the list – it would be good to talk to him about the reasons for this. (There is a repetition here so this might be worth discussing too.)
The problem offers opportunities for children to use different methods	Clip "Eggs3.wmv" Here the teacher draws attention to the children's different ways of representing the problem (drawing sweets, using numerals, drawing dots, writing number sentences), emphasising why each is helpful. Interestingly, some children chose to opt for a different way following this discussion. Image "EggsB.gif" This pupil has chosen to represent five sweets in the "quincunx" arrangement, like that on a dice. Perhaps this is to make subsequent counting easier?
Eggs in Baskets offers opportunities to broaden students' problem-solving skills	Clip "Eggs4.wmv" Having been shown there are two sweets in the first bag and three in the second bag, the children talk about whether they need to see the number of sweets in the third bag. One says, "2 add 3 equals 5, add 1 equals 6." "So I don't need to X-ray the last one?" Another pupil responds, "You do! Just to see …" "You'd like to check it using that?" "It's still going to be 1." This highlights the fact that it may be satisfying to check that our conjectures are true before moving on. See also image "EggsA.gif"

Appendix 5	for Activity 1.1	a copy for each teacher of either appendices 3&4, or of appendices 5&6

This is a classroom task from the NRICH website.
Notes about the task, including a projectable, online, interactive presentation of the task are available from: tinyurl.com/5fmlao

GOT IT

GOT IT is an adding game for two. You can play against the computer or with a friend.

Start with the GOT IT target 23.

The first player chooses a whole number from 1 to 4.

Players take turns to add a whole number from 1 to 4 to the running total.

The player who hits the target of 23 wins the game.

To change the game, choose a new GOT IT! target or a new range of numbers to add on.

Appendix 6	for Activity 1.1	a copy for each teacher of either appendices 3&4, or of appendices 5&6

RICH TASK TEMPLATE – GOT IT

WHY IS *GOT IT* A RICH TASK?

Step into a problem even when the route to a solution is unclear (see definition of "a problem" below), getting started and exploring is made accessible to pupils of wide ranging abilities.	The game is an engaging one and pupils are often motivated to find strategies in order to beat the computer or, if you do not have access to the interactivity, to beat the teacher.
Pose as well as solve problems, make conjectures	Pupils often extend the problem to different target numbers and a different range of numbers quite naturally. Other extensions include choosing a range of numbers that do not start at 1.
Work at a range of levels	Some pupils are excited to discover that the person reaching 18 first will win. You can simplify the starting point further with a lower target number and smaller range of numbers. At the highest level the generalisation to any target, any range requires high-level thinking and analytical skills
Extend knowledge or apply knowledge in new contexts	This is a different and engaging context to meet and engage with mathematics
Allow for different methods	It is interesting to see the different ways in which pupils come to an understanding of why their strategy works.
Offer opportunities to broaden students' problem-solving skills	Working backwards is a very useful skill in this case. Generalising results to any target and range and identifying the exceptions.
Deepen and broaden mathematical content knowledge	In this task pupils are being asked to recognise and explain patterns and relationships, conjecture, generalise and predict. At the highest levels they should justify their generalisations using convincing arguments and proofs.
Have potential to reveal underlying principles or make connections between areas of mathematics	For example the unexpected connection with factors and multiples
Include intriguing contexts	It is not obvious that employing some mathematics will guarantee that you can always win.
Offer opportunities to observe other people being mathematical or the role of mathematics within cultural settings	Pupils challenging the teacher or computer and explaining what they will do next and why whilst others observe and listen

Appendix 7	for Activity 1.2	a copy for each pair of teachers (4 pages)

HOTS1

These cards contain some lower-order questions and, focusing on the same mathematical topic, some more challenging questions - ones that require higher-order thinking skills. Cut them out and pair them up.

1. Fractions

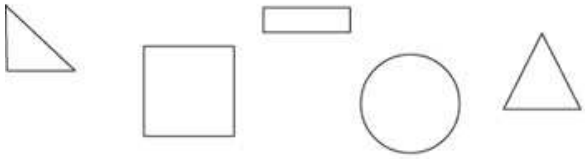
What is half of 6?

What is half of 10?

What is half of 2?

2. Triangles

Which of these shapes are triangles?



3. Fair Feast

Here is a picnic that Chris and Michael are going to share equally:

Can you tell what each of them will have?

4. Grab it!

Play on a blank 100 grid with a partner. Take turns to choose a number. If your number can be divided exactly by 2, score 2 points. If it can be divided exactly by 3, score 3 points and so on. (You can decide whether or not to count 1 and the number itself.)

What are good numbers to pick? Why?

What's the best number to pick?

What are poor numbers to pick? Why?

5. Take Away

Work out the following take away (subtraction) sums:

654 -132	780 -444	323 -178	.....etc

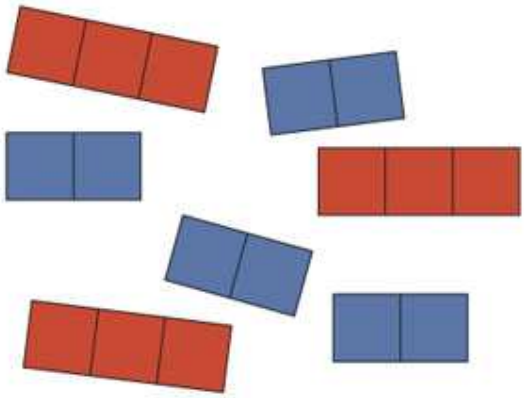
6. Hard or Easy?

Look at the take away sums. Find the easiest, the hardest, and three which are not hard or easy. Do them and write down (or say) why you've chosen these five sums.

654 -132	780 -444	323 -178	.....etc

7. Making Sticks

Kimie and Sebastian were making sticks from interlocking cubes. Kimie made blue sticks 2 cubes long. Sebastian made red sticks 3 cubes long. They both made a lot of sticks.



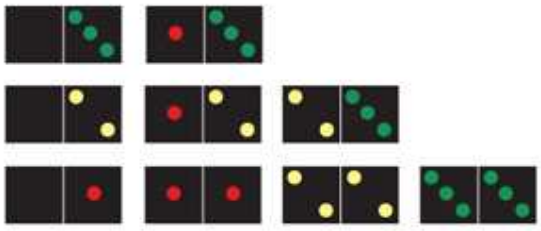
Kimie put her blue sticks end to end in a long line. Sebastian put his red sticks end to end in a line underneath Kimie's.

Can they make their lines the same length? How many sticks could Kimie use? How many would Sebastian put down? How long is the line altogether?

Can they make any other lines?

8. Domino Sorting

Here are some dominoes taken out of a full set:



Sort them into two groups, one with an odd number of spots and one with an even number of spots.

Do you have any dominoes left over? Why, or why not?

Now put the dominoes into pairs. The number of spots on each pair of dominoes must make a total of 5.

How many pairs can you make?

Which dominoes are left over?

Can you pair them up in any different ways so that each pair adds up to 5?

Which dominoes are left over now?

Are there any dominoes which are always left over?

Can you explain why?

9. Seven Sticks

Explore the triangles that can be made with seven sticks of the same length.

10. Near Doubles

Add these near doubles

20 + 2155 + 5648 + 50 ...etc

11. The Hundred Game

This game is for two players. You need ten cards with the digits 0 to 9 on them. It might also be useful to have a two pieces of paper or card with two boxes drawn on them to represent a two-digit number.

Turn the cards face down and mix them up. The aim of the game is to make the closest number to 100. Each player takes one card to start with and decides whether that is the units or tens digit of their number and places it on their paper in front of them. Each player then takes a second card which becomes the missing digit of their two-digit number. The winner is the player whose number is closer to 100. You could have a points system so that the player with the closer number scores 1 point and then play first to 10.

What strategies do you have for winning?

12. Squares

Plot the three points listed below and then find the co-ordinates of the fourth point that is needed to complete a square:

(a) (2,2) (4,2) (2,4)

(b) (5,10) (9,10) (9,6)

(c) (4,5) (3,6) (2,5)

(d) (5,5) (4,8) (7,9)

(e) etc.

13. Stringy Quads

You need a group of four people holding a loop of string.

Make a quadrilateral with one line of symmetry.

Make a quadrilateral with two lines of symmetry.

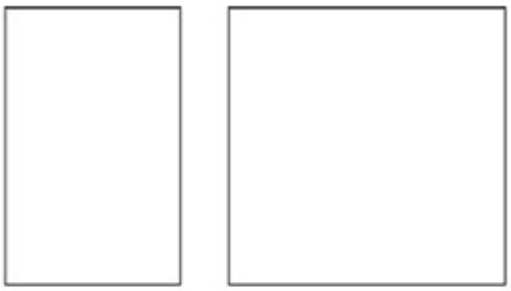
Make a quadrilateral with three lines of symmetry.

Make a quadrilateral with four lines of symmetry.

What quadrilateral haven't you made?

14. Symmetry

Draw all the lines of symmetry on these quadrilaterals.



etc

15. Addition

What is:

5+4?

3+9?

2+5?

3+3?

16. U Two

You need a 1-50 number grid and a partner. Take turns to draw a 5 square U shape on the grid. Add up the two biggest numbers in your U. Keep going until you can't fit any more Us on the grid, adding on your score each time. The winner has the bigger score. Your U could be upside down, or on its side.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

17. Sharing

Do the following division (sharing) sums.

45÷5 48÷6 28÷7 ...etc

18. Place Value

Which of these numbers is bigger:

78 or 87?

92 or 91?

99 or 101?

19. Square It

With a partner take it in turns to mark any spot on a square dotty grid (you should use different colours).

The winner is the first to have four marks that can be joined by straight lines to form a square.

Squares can be of any size and can be tilted.

20. Multiples

List the numbers between 1 and 30 that are:

(a) in the two times table

(b) in the three times table.

Appendix 8	for Activity 1.2	a copy for each pair of teachers

HOTS2

Encouraging Higher Order Thinking Skills (HOTS)

Instead of asking different questions, we can change the ones we usually ask. How can you adapt ordinary maths questions so that they promote HOTS? Here are four key strategies that will help you to increase the challenge of standard questions in the classroom.

.

A. Here's the answer, what could the question be?

Instead of: 3+3, 4+3, 5+3, 6+3.....

Ask: The answer is 8; what could the adding up sum be?

Instead of: What is the area of a rectangle which measures 4cm by 6cm?

Ask: If the area of a rectangle is 24cm^2 what could its measurements be?

Lists of practice questions and closed questions can immediately be made more challenging in this way, and this change allows children to show what they know and can do. You may well be surprised by the quality of their work! Some children will work systematically to produce their responses; this indicates that they have analysed the numerical structure.

Make up some examples of your own.

B. Make up your own

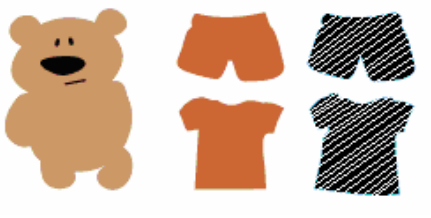
Instead of: 456 - 354, 1008 - 783, 6666 - 3333, 7065 - 4999, ...

Ask: Choose the easiest and hardest subtraction sums, work them out, then make up an easy and hard example for someone else, saying why you think there are easy and hard.

Choosing requires analysis, making up new questions requires synthesis, and sharing and discussing with another requires evaluation.

Can you make up some similar examples involving other operations? How about other mathematical topics such as space and shape?

C. What if?



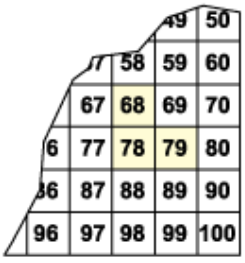
Instead of: Find the different ways you can dress the teddy.

Ask: What if there were two teddies?

What if there were two hats as well?

What if there were three T-shirts?

What if... ?



Instead of: Put the L on the grid so that the sum of the squares it covers is 225.

Ask: What if the sum is different?

What if the shape is not an L?

What if the grid is the two times table?

What if...?

Offering choice often increases children's motivation and hence engagement in a task. They have to understand the structure of the question in order to make sensible 'what if' suggestions. They will need to identify what aspects of the problem can be varied - analysis and synthesis.

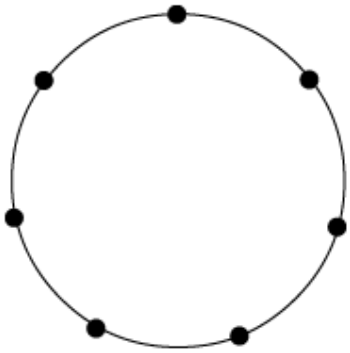
Look at questions you have recently given your pupils to do. Can you think of some "what if" questions.

How would you encourage pupils to come up with "what if" questions of their own?

D. All answers

Instead of: Make a symmetrical necklace with these beads.

Ask: Make another... make another... how many can you make? How do you know you've got them all?



Instead of: Make a triangle by joining three dots.

Ask: Make another... make another... how many can you make? How do you know you've got them all?

Appendix 9	for Activity 1.3 & 1.4	a copy for each pair of teachers

BLOOM'S DESCRIPTORS

Cut the cards out and put them in a line to reflect the order of development, complexity and demand (which represent the higher order thinking skills).

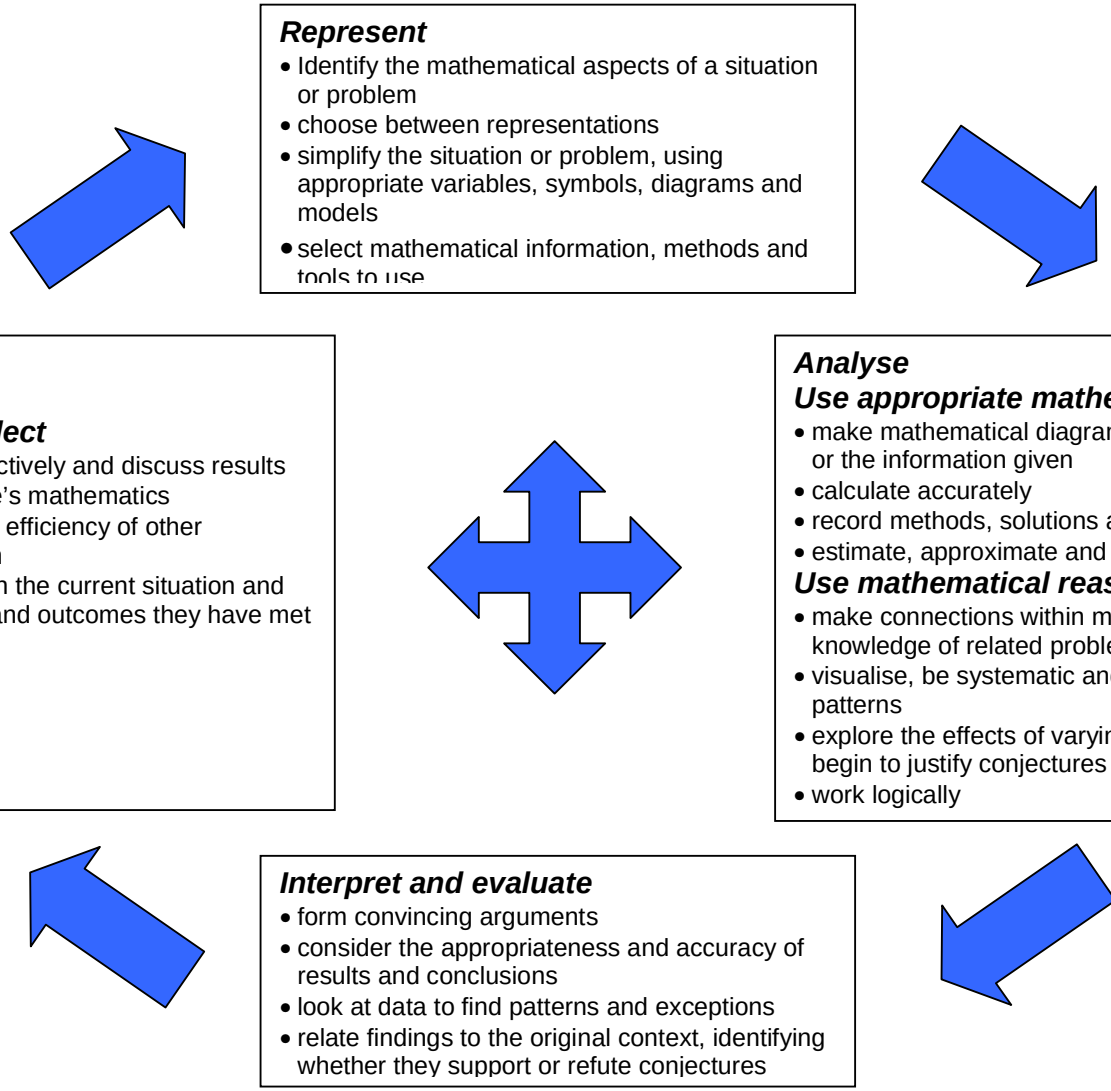
Analysis seeing pattern organization of parts recognition of hidden meanings identification of components **Question Cues:** analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer	***Evaluation*** compare and discriminate between ideas assess value of theories, presentations make choices based on reasoned argument verify value of evidence recognize subjectivity **Question Cues:** assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize
Application use information use methods, concepts, theories in new situations solve problems using required skills or knowledge **Questions Cues:** apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover	***Knowledge*** observation and recall of information knowledge of dates, events, places knowledge of major ideas mastery of subject matter **Question Cues:** list, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
Comprehension understanding information grasp meaning translate knowledge into new context interpret facts, compare, contrast order, group, infer causes predict consequences **Question Cues:** summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend	***Synthesis*** use old ideas to create new ones generalize from given facts relate knowledge from several areas predict, draw conclusions **Question Cues:** combine, integrate, modify, rearrange, substitute, plan, create, design, invent, what if?, compose, formulate, prepare, generalise, rewrite

Appendix 10	for Activity 1.4	a copy for each pair of teachers

Represent • Identify the mathematical aspects of a situation or problem • choose between representations • simplify the situation or problem, using appropriate variables, symbols, diagrams and models • select mathematical information, methods and tools to use.	
Analyse **Use appropriate mathematical procedures** • make mathematical diagrams that represent a situation or the information given • calculate accurately • record methods, solutions and conclusions • estimate, approximate and check working	**Analyse** **Use mathematical reasoning** • make connections within mathematics and use knowledge of related problems • visualize, be systematic, and identify and classify patterns • explore the effects of varying values and make and begin to justify conjectures and generalisations • work logically
Interpret and evaluate • form convincing arguments • consider the appropriateness and accuracy of results and conclusions • look at data to find patterns and exceptions • relate findings to the original context, identifying whether they support or refute conjectures	**Communicate and reflect** • communicate findings effectively and discuss results • engage with someone else's mathematics • consider the elegance and efficiency of other approaches to the problem • make connections between the current situation and outcomes, and situations and outcomes they have met before

Appendix 11	for Activity 1.4	a copy for each teacher

The Problem-Solving Cycle



Appendix 12	for Activity 1.4 & 2.1	a copy for each pair of teachers

RICH TASK CARDS

step into a problem even when the route to a solution is unclear, getting started and exploring is made accessible to pupils of wide ranging abilities	pose as well as solve problems, make conjectures
work at a range of levels	extend knowledge or apply knowledge in new contexts
have opportunities to broaden their problem-solving skills	deepen and broaden mathematical content knowledge
potentially reveal underlying principles or make connections between areas of mathematics	observe other people being mathematical or see the role of mathematics within cultural settings
experience intriguing contexts	employ different methods

Appendix 13	for Activity 1.5	a copy for each pair of teachers (2 pages)

PROGRESSION CARDS

begin to develop own ways of recording	put the problem into their own words
begin to look for patterns in results as they work and use them to find other possible outcomes	recognise information that is important to solving the problem, determine what is missing and develop lines of enquiry
begin to organise their work and check results	respond to 'What if?' questions
begin to understand and use formulae and symbols to represent problems	review their work and approaches
begin to work in an organised way from the start	review their work and reasoning,
break a several-step problem or investigation into simpler steps	search for a solution by trying out ideas of their own
check answers and ensure solutions make sense in the context of the problem	select the mathematics they use in a wider range of classroom activities, e.g.
check as they work, spotting and correcting errors and reviewing methods	show understanding of situations by describing them mathematically using symbols, words and diagrams
check their methods and justify answers	talk about their findings by referring to their written work
check their work and make appropriate corrections, e.g. decide that two numbers less than 100 cannot give a total more than 200 and correct the addition	try different approaches and find ways of overcoming difficulties that arise when they are solving problems
choose their own equipment appropriate to the task, including calculators	understand a general statement by finding particular examples that match it
consider appropriate units	use and interpret mathematical symbols and diagrams
consider efficient methods, relating problems to previous experiences	use appropriate mathematical vocabulary
decide how best to represent conclusions, using appropriate recording	use classroom discussions to break into a problem, recognising similarities to previous work

develop an organised approach as they get into recording their work on a problem	use examples and counter-examples to justify conclusions
discuss their mathematical work and begin to explain their thinking, e.g.	use related vocabulary accurately
draw simple conclusions of their own and give an explanation of their reasoning	Use their own strategies within mathematics and in applying mathematics to practical context
explain and justify their methods and solution	when they have solved a problem, pose a similar problem for a partner
Identify and obtain necessary information to carry through a task and solve mathematical problems	With support adopt a suggested model or systematic approach
identify more complex patterns, making generalisations in words and begin to express generalisations using symbolic notation	With support begin to appreciate the need to record and develop their own methods of recording
identify patterns as they work and form their own generalisations/rules in words	With support describe the strategies and methods they use in their work
make a generalisation with the assistance of probing questions and prompts	With support find a starting point, identifying key facts/relevant information
make connections to previous work	With support listen to others' explanations, try to make sense of them, compare.… evaluate…
make their own suggestions of ways to tackle a range of problems	With support make connections and apply their knowledge to similar situations
organise their work from the outset, looking for ways to record systematically	With support move between different representations of a problem e.g. a situation described in words, a diagram etc.
organise written work, e.g. record results in order	With support test a statement such as 'The number 12 ends with a 2 so 12 sweets can't be shared equally by 3 children'
pose and answer questions related to a problem	With support use apparatus, diagrams, role play, etc. to represent and clarify a problem
predict what comes next in a simple number, shape or spatial pattern or sequence and give reasons for their opinions	With support use pictures, diagrams and symbols to communicate their thinking, or demonstrate a solution or process

Appendix 14	for Activity 1.5	a copy for each pair of teachers (2 pages)

Progression Card List

Based on the National Strategy's Primary Framework Assessment Guidelines for mathematics L2, L3, L4, L5 2008

L2	1.0x	select the mathematics they use in some classroom activities e.g. with support	PS
L2	1.1	With support find a starting point, identifying key facts/relevant information	PS
L2	1.2	With support use apparatus, diagrams, role play, etc. to represent and clarify a problem	PS
L2	1.3	With support move between different representations of a problem e.g. a situation described in words, a diagram etc.	PS
L2	1.4	With support adopt a suggested model or systematic approach	PS
L2	1.5	With support make connections and apply their knowledge to similar situations	PS
L2	2.0x	discuss their work using mathematical language, e.g. with support	C
L2	2.1	With support describe the strategies and methods they use in their work	C
L2	2.2	With support listen to others' explanations, try to make sense of them, compare…. evaluate…	C
L2	3.0x	begin to represent their work using symbols and simple diagrams, e.g. with support	C
L2	3.1	With support use pictures, diagrams and symbols to communicate their thinking, or demonstrate a solution or process	C
L2	3.2	With support begin to appreciate the need to record and develop their own methods of recording	C
L2	4.0x	explain why an answer is correct, e.g. with support	R
L2	4.1	With support test a statement such as 'The number 12 ends with a 2 so 12 sweets can't be shared equally by 3 children'	R
L2	5.0	predict what comes next in a simple number, shape or spatial pattern or sequence and give reasons for their opinions	R
L3	1.0	select the mathematics they use in a wider range of classroom activities, e.g.	PS
L3	1.1	use classroom discussions to break into a problem, recognising similarities to previous work	PS
L3	1.2	put the problem into their own words	PS
L3	1.3	choose their own equipment appropriate to the task, including calculators	PS
L3	2.0	try different approaches and find ways of overcoming difficulties that arise when they are solving problems	PS
L3	2.1	check their work and make appropriate corrections, e.g. decide that two numbers less than 100 cannot give a total more than 200 and correct the addition	PS
L3	2.2	begin to look for patterns in results as they work and use them to find other possible outcomes	PS
L3	3.0	begin to organise their work and check results	C
L3	3.1	begin to develop own ways of recording	C
L3	3.2	develop an organised approach as they get into recording their work on a problem	C
L3	4.0	discuss their mathematical work and begin to explain their thinking, e.g.	C
L3	4.1	use appropriate mathematical vocabulary	C
L3	4.2	talk about their findings by referring to their written work	C
L3	5.0	use and interpret mathematical symbols and diagrams	C
L3	6.0	understand a general statement by finding particular examples that match it	R
L3	6.1	make a generalisation with the assistance of probing questions and prompts	R
L3	7.0	review their work and reasoning,	R
L3	7.1	respond to 'What if?' questions	R
L3	7.2	when they have solved a problem, pose a similar problem for a partner	R
L4	1.0x	develop own strategies for solving problems, e.g.	PS
L4	1.1	make their own suggestions of ways to tackle a range of problems	PS
L4	1.2	make connections to previous work	PS
L4	1.3	pose and answer questions related to a problem	PS
L4	1.4	check answers and ensure solutions make sense in the context of the problem	PS
L4	1.5	review their work and approaches	PS
L4	2.0	Use their own strategies within mathematics and in applying mathematics to practical context	PS
L4	3.0x	present information and results in a clear and organised way, e.g.	C
L4	3.1	organise written work, e.g. record results in order	C
L4	3.2	begin to work in an organised way from the start	C
L4	3.3	consider appropriate units	C
L4	3.4	use related vocabulary accurately	C
L4	4.0	search for a solution by trying out ideas of their own	R
L4	4.1	check their methods and justify answers	R
L4	4.2	identify patterns as they work and form their own generalisations/rules in words	R

L5	1.0	Identify and obtain necessary information to carry through a task and solve mathematical problems	PS
L5	1.1	recognise information that is important to solving the problem, determine what is missing and develop lines of enquiry	PS
L5	1.2	break a several-step problem or investigation into simpler steps	PS
L5	1.3	consider efficient methods, relating problems to previous experiences	PS
L5	2.0x	check results, considering whether these are reasonable, e.g.	PS
L5	2.1	check as they work, spotting and correcting errors and reviewing methods	PS
L5	3.0	solve word problems and investigations from a range of contexts	PS
L5	3.0	show understanding of situations by describing them mathematically using symbols, words and diagrams	C
L5	3.1	organise their work from the outset, looking for ways to record systematically	C
L5	3.2	decide how best to represent conclusions, using appropriate recording	C
L5	3.3	begin to understand and use formulae and symbols to represent problems	C
L5	4.0	draw simple conclusions of their own and give an explanation of their reasoning	R
L5	4.1	explain and justify their methods and solution	R
L5	4.2	identify more complex patterns, making generalisations in words and begin to express generalisations using symbolic notation	R
L5	4.3	use examples and counter-examples to justify conclusions	R

Appendix 15	for Activity 2.1	a copy for each teacher (2 pages)

A RICH TASK – WHAT TEACHERS CAN DO

Aspects of a rich task	Ideas for teacher support
Step into a problem even when the route to a solution is unclear, getting started and exploring is made accessible to pupils of wide ranging abilities.	Selecting appropriate tasks and problems for example – those with a low threshold and a high ceiling. Asking pupils to spend a little time on their own then working in pairs and then sharing ideas about what the problem is about and how to get started Think ⇨ Pair ⇨ Share Encouraging some general exploration of the situation before pinning things down Considering and sharing different ways of representing the information Thinking about things like this you have seen before.
Pose as well as solve problems, make conjectures	It is a challenge for pupils to pose their own problems so a first step is to model asking "what if" questions yourself. Encourage learners to think about the things they can vary in a problem and to conjecture about the effect of any variation. At the end of a problem ask "what next?" or "If we had time what might we do next?" Highlight occasions where pupils do pose their own problems and share them with the group. Put unanswered questions and conjectures on a board. Use a "conjecture board". When pupils come up with a conjecture they write it up and get others to consider it and either prove it or find a counter example
Work at a range of levels	Encouraging and discussing different ways of tackling a problem. Interpreting and evaluating findings can offer opportunities to work at a range of levels. Think about problems with open starting points, open middles and open ends – these all contribute to allowing pupils to work at different levels. Generalisation enables extension and the use of algebra can extend problems. Reflect on the algebra, when it is used, and how it represents underpinning structure of a problem. For example: "Why does ….. generate a Fibonacci sequence?"
Extend knowledge or apply knowledge in new contexts	Set problems that offer scope to extend knowledge or which are set in new contexts. Ask questions of learners that encourage them to make connections: "Have you done something before that was similar?" "What mathematics is in this problem?"
Allow for different methods	Encourage a range of representations at the start of the work. Discuss ideas for different approaches. Discuss different approaches, their effectiveness and efficiency at the end of the work. Value different approaches as representing learners' different understandings and levels of confidence. Realise that methods used often reflect learners' progress, areas of strength and weaknesses.

Aspects of a rich task	Ideas for teacher support
Offer opportunities to broaden students' problem-solving skills	Talk about what a pupil is doing. For example: "How will you collect the data?" "Was that a good method, are there other ways that might have been more efficient?". "Can you be more systematic?" "Can you generalise?"
Deepen and broaden mathematical content knowledge	Use problems that offer challenging contexts in which can help develop content knowledge Highlight the mixture of skills pupils are bringing to bear of problems: "In this problem you needed to be able to… in order to …." Ask pupils what mathematics they used to tackle the problem, new things they have learnt and what they feel more confident about.
Have potential to reveal underlying principles or make connections between areas of mathematics	Problems like this might not be as engaging at first sight – their fascination comes from the patterns or ideas they reveal as you work on them. For example: The relationship between square and triangular numbers might come out of work on triangular numbers. Games or problems that have the same underpinning mathematics (e.g. nim or variations on noughts and crosses)
Include intriguing contexts	Use games or challenges. Use problems that reveal interesting patterns. Identify mathematics in unfamiliar settings. When you notice some mathematics why not draw attention to it and use it. For example the sun shining through the window, arrangements of the desks, work on sports day such as laying out the track and recording results. When you see something intriguing in some mathematics draw pupils attention to it. For example, an unexpected pattern in geometry or arithmetic that needs to be explained. That two shapes with the same volume look completely different. Make a note on the board and ask pupils to think about it and return to it at "odd moments" over a period of time.
Offer opportunities to observe other people being mathematical or the role of mathematics within cultural settings	Model being stuck sometimes. Allow pupils to ask and work on problems you do not know the answer to and say so. "We will find out about this together" Use video and films related to mathematics being used or which put mathematics in historical and cultural contests. For example, when tackling a problem involving a Fibonacci sequence show some examples of its occurrence in the world around us. When talking about being stuck discuss what mathematicians do. When doing work on time look at how this has been measured in the past.

Appendix 16	for Activity 2.1 & 2.2 & 4.1	a copy for each teacher (2 pages)

WHAT TEACHERS DO MASTER SHEET

Aspect of a Rich Task	What pupils could do	What teachers might do
step into a problem even when the route to a solution is unclear, getting started and exploring is made accessible to pupils of wide ranging abilities.		
pose as well as solve problems, make conjectures		
work at a range of levels		
extend knowledge or apply knowledge in new contexts		
allow for different methods		

offer opportunities to broaden students' problem-solving skills		
deepen and broaden mathematical content knowledge		
have potential to reveal underlying principles or make connections between areas of mathematics		
include intriguing contexts		
offer opportunities to observe other people being mathematical or the role of mathematics within cultural settings		

Appendix 17	for Activity 2.1	a copy for each pair of teachers (4 pages)

MAGIC Vs – WHAT TEACHERS DO

Text in italics shows examples from a single lesson, with associated video-clips and images. This is not intended to be a model lesson.

Aspect of a Rich Task	What pupils might do	What teachers could do
step into a problem even when the route to a solution is unclear (see definition of "a problem" below), getting started and exploring is made accessible to pupils of wide ranging abilities.	By starting with the challenge of making a "Magic V" pupils wide ranging abilities can get into this problem. Sharing early findings can move the challenge on to finding all the solutions.	Allow pupils to engage with the attributes of a Magic V by identifying similarities and differences between a Magic V and a non-magic V. *Pupils are given a very open task to work on initially ("what questions could we ask about this?) and the teacher "funnels" the suggestions to focus on particular ideas* *Clips "MagicV1.wmv", "MagicV2.wmv", "MagicV3.wmv" and image "MagicVA.jpg"* Give pupils time to work on the problem on their own Use 'Think – pair – share' Ideas about what you notice and then conjectures pupils might make Share different ways of recording
pose as well as solve problems, make conjectures	The task lends itself readily to pupils posing their own problems and making conjectures, for example: Why is the number at the bottom always odd? *A pupil conjectures that this is the case if you have 3 odd numbers and 2 even numbers, but that if you have 3 even numbers and two odd ones then the bottom number will be even. Clip "MagicV4.wmv"* Will it always be odd? I think the number at the bottom is odd because there are more odds in the numbers 1-5 than evens…. If I can find two pairs of numbers that add to the same total to go on the two "arms" of the V then it doesn't matter which number goes at the bottom. *A pupil tries to find an example that satisfies this conjecture:* *Clip "MagicV5.wmv"*	Ask questions such as: "What do you notice?" "Can we justify that" Write conjectures on the board. Encourage the whole group to work on an idea posed by one of their class.

	I think that opposite numbers in the arms of the V will have the same difference. *See the clip "MagicV6.wmv" for further explanation of this. The pupils in this clip decide, wrongly, that this conjecture is incorrect.*	
work at a range of levels	Pupils who do not readily work in a systematic way can gain insights into the value of being systematic and organised in their thinking. Some pupils are able to see why odd numbers must go at the bottom and the most able are able to develop convincing arguments for what will happen for any V. *A pupils explains that he tried putting an even number at the bottom but then found he was left with three odds and an even which don't make an even total and therefore can't be split equally between the two arms. This "proof by contradiction" is a higher-order skill that children rarely use. The clip "MagicV7.wmv" exemplifies this. [This clip also exemplifies the value of giving pupils thinking time.]* New knowledge can then be applied to different scenarios such as crosses or, more challenging H's. *The clip "MagicV8.wmv" shows a pupil trying a Magic Cross.*	Encourage pupils to write down findings; the teacher could demonstrate good recording methods to the class, or could share ideas that the pupils have developed. Provide materials (such as cards) for pupils to manipulate, so they can have greater confidence to try some ideas rather than aiming for an immediately correct solution. *See the clip "MagicV6.wmv" for an example of this.* [Discussion point: should we allow <u>all</u> children to choose whether or not to use materials such as cards, or only issue them to certain pupils?] Have ideas for extending the problem ready but try to encourage pupils to come up with ideas of their own with you helping them to select "fruitful" routes Encourage able pupils to generalise – be ready with counter examples to get them rethinking. For example "always an odd at the bottom" does not work with the numbers 2,3,4,5,6 so set them the problem with different numbers. Ask: "What are the variables/what can we change?"
extend knowledge or apply knowledge in new contexts	This does not require the application of high level content knowledge but this means that proof and convincing arguments associated with the setting can be shared and understood. I have often seen generalisations produced (for example – if there are more odds an odd goes at the bottom) that can be refuted. Refutation is a higher order thinking skill that pupils rarely employ rigorously.	Ask pupils to "prove it" Or ask "how do you know that will always be the case" When tackling problems in new contexts (such as larger Vs, crosses or, more challenging H's). Ask pupils not only to solve the problems but to describe what strategies they re-employed. "What things worked and what didn't?" "What was the same and what different"

Aspect of a Rich Task	What pupils might do	What teachers could do
allow for different methods	This task opens up a wide range of methods for finding solutions and offers room for much discussion. *A pupil asks a friend to explain their idea with greater clarity: clip "MagicV9.wmv".* *An alternative method is described in clip "MagicV10.wmv" and image "MagicVB.jpg". The five numbers add to a total of 15, so once one number is chosen to go at the bottom of the V (in this example, 5), the rest (10) must be split equally between the two "arms" of the V.*	Share different methods and discuss efficiency and effectiveness. An efficient method is only useful if you can use it. For example: the sum of the numbers 1 – 5 is 15 to share equally in the two arms an odd goes at the bottom and the rest is shared so: 15-5 = 10, then 10/2 is 5. A total of 5 in each arm means 1+4 and 2+3. 15-3 = 12, then 12/2 is 6. A total of 6 in each arm means 1+5 and 2+4… Find all the solutions to V with 2,3,4,5,6 in your head… *In the clip "MagicV11.wmv" the teacher draws attention to the efficient way that one group worked. They shared out the task so they all tried different possibilities.*
offer opportunities to broaden students' problem-solving skills	Being systematic is at the core of this problem. *This child demonstrates all the possible arrangements for a certain magic total: clip "MagicV12.wmv" and image "MagicVC.jpg"* *Another child then explains how she uses the previous clip to work out how many Magic Vs there are altogether: clip "MagicV13.wmv"* Identifying pattern and generalisation then enables similar problems to be tackled more efficiently (Have you seen something like this before?)	Share efficient and systematic recording methods and approaches to the problem. Ask pupils if they would tackle a similar problem in the same or a different way next time. Why? "Where else has it been useful to be systematic in this way?"
deepen and broaden mathematical content knowledge	In this task pupils are being asked to recognise and explain patterns and relationships, conjecture, generalise and predict. At the highest levels they should justify their generalisations using convincing arguments and proofs.	Less able pupils will be honing their number bond and mental calculation skills. They can be encouraged to look at different starting numbers and different sized V's. Use pieces of paper to layout and try things out. Establishing rules for adding odd and even numbers including simple proofs (picture proofs). For example odd+odd=even might look like: :::::. + ':::::: = ::::::::::::: More able pupils can be encouraged to generalise rules and assess peers on the rigour of their proofs.

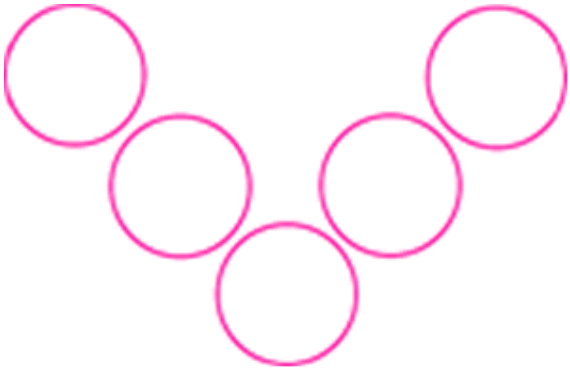
Aspect of a Rich Task	What pupils might do	What teachers could do
have potential to reveal underlying principles or make connections between areas of mathematics	A powerful underlying concept here is the relationships between even and odd numbers and sums of consecutive numbers.	See above re odds and evens. That you can add, subtract, multiply or divide numbers in a Magic V and it will still work. Although a Magic T looks the same, if the trunk of the T is longer than the arms it does not work – why? Where else is it useful to be systematic? Where have we worked before where we have listed all possible outcomes? Dipping games rely on odds and evens – can you arrange to make sure that a particular person is left at the end.
include intriguing contexts	Pupils are intrigued by identifying efficient and labour – saving strategies	Discussing efficient strategies For example the method described above works because it is efficient and there is a clear structure. How about other methods, do they generalise? "Why do you like this method or someone else's method more?"
offer opportunities to observe other people being mathematical or the role of mathematics within cultural settings	As a teacher you can model efficient techniques for solution to stimulate discussion… "Now this is what I call efficient"… followed by modelling the process. I have also found pupils seeing patterns in underpinning mathematics that I had not noticed and it is good for pupils to see you having to struggle to understand someone else's idea.	When pupils suggest ideas and strategies try to take on the role of learner asking questions such as: "Why did you do that?" "What should I do if" "Would it work if I…?" - even if you think you know. *In clip "MagicV14.wmv" and image "MagicVD.jpg" the teacher explicitly draws attention to the use of "proof by contradiction" as a powerful way to approach this problem.* *Clip "MagicV15.wmv" shows the teacher highlighting how findings from Magic Vs can be applied to other letter shapes.*

Appendix 18	for Activity 2.1	a copy for each teacher

This is a classroom task from the NRICH website.
Notes about the task, including more resources are available from: tinyurl.com/5vte3f

Magic Vs

Place each of the numbers 1 to 5 in the V shape below so that the two arms of the V have the same total.



How many different possibilities are there?
What do you notice about all the solutions you find?

Can you explain what you see?

Can you convince someone that you have all the solutions?

What happens if we use the numbers from 2 to 6? From 12 to 16? From 37 to 41? From 103 to 107?

What can you discover about a V that has arms of length 4 using the numbers 1-7?

Appendix 19	for Activity 3	a copy for each teacher (or use it on the website) (4 pages)

NRICH www.nrich.maths.org problems linked to the Framework for teaching mathematics in Foundation, Year 1 and Year 2

N.B. This is work in progress

	Foundation	Year 1	Year 2
	Strand 1 - Using and Applying		
	Use developing mathematical ideas and methods to solve practical problems	Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change'	Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence **NRICH:** Eggs in Baskets **NRICH:** The Brown Family
	Match sets of objects to numerals that represent the number of objects	Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem **NRICH:** Birthday Cakes **NRICH:** The Amazing Splitting Plant
	Sort objects, making choices and justifying decisions	Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures	Follow a line of enquiry; answer questions by choosing and using suitable equipment and selecting, organising and presenting information in lists, tables and simple diagrams
	Talk about, recognise and recreate simple patterns	Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions	Describe patterns and relationships involving numbers or shapes, make predictions and test these with examples **NRICH:** Caterpillars
	Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices	Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures	Present solutions to puzzles and problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences
	Strand 2 - Counting and Understanding Number		
	Say and use number names in order in familiar contexts	Count reliably at least 20 objects, recognising that when rearranged the number of objects stays the same; estimate a number of objects that can be checked by counting **NRICH:** Making Sticks **NRICH:** Biscuit Decorations	Read and write two-digit and three-digit numbers in figures and words; describe and extend number sequences and recognise odd and even numbers **NRICH:** Ring a Ring of Numbers **NRICH:** Domino Sequences **NRICH:** Domino Number Patterns **NRICH:** Next Domino

Know that numbers identify how many objects are in a set	Compare and order numbers, using the related vocabulary; use the equals (equals) sign	Count up to 100 objects by grouping them and counting in tens, fives or twos; explain what each digit in a two-digit number represents, including numbers where 0 is a place holder; partition two-digit numbers in different ways, including into multiples of 10 and 1 **NRICH:** Grouping Goodies
Count reliably up to 10 everyday objects	Read and write numerals from 0 to 20, then beyond; use knowledge of place value to position these numbers on a number track and number line **NRICH:** Tug of War	Order two-digit numbers and position them on a number line; use the greater than (greater than) and less than (less than) signs **NRICH:** 100 Square Jigsaw
Estimate how many objects they can see and check by counting	Say the number that is 1 more or less than any given number, and 10 more or less for multiples of 10	Estimate a number of objects; round two-digit numbers to the nearest 10
Count aloud in ones, twos, fives or tens **NRICH:** Incey Wincey Spider	Use the vocabulary of halves and quarters in context	Find one half, one quarter and three quarters of shapes and sets of objects **NRICH:** Halving **NRICH:** Happy Halving
Use language such as 'more' or 'less' to compare two numbers		
Use ordinal numbers in different contexts		
Recognise numerals 1 to 9		
Strand 3 – Knowing and Using Number Facts		
Observe number relationships and patterns in the environment and use these to derive facts	Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts **NRICH:** Cuisenaire Environment **NRICH:** Domino Sorting	Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100 **NRICH:** Weighted Numbers **NRICH:** Number Balance
Find one more or one less than a number from 1 to 10	Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple **NRICH:** Are You Well Balanced? **NRICH:** Buzzy Bee	Understand that halving is the inverse of doubling and derive and recall doubles of all numbers to 20, and the corresponding halves **NRICH:** The Tomato and the Bean
Select two groups of objects to make a given total of objects	Recall the doubles of all numbers to at least 10 **NRICH:** Magic Plant	Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; Recognize multiples of 2, 5 and 10 **NRICH:** Clapping Times **NRICH:** Lots of Lollies
		Use knowledge of number facts and operations to estimate and check answers to calculations

Strand 4 – Calculating		
Begin to relate addition to combining two groups of objects and subtraction to 'taking away'	Relate addition to counting on; recognize that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number **NRICH:** Number Lines **NRICH:** Getting the Balance **NRICH:** Ladybirds in the Garden	Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers **NRICH:** Butterfly Flowers **NRICH:** Number Round Up
In practical activities and discussion begin to use the vocabulary involved in adding and subtracting	Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number	Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences **NRICH:** Secret Number
Count repeated groups of the same size	Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences **NRICH:** 2,4,6,8	Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders **NRICH:** Share Bears
Share objects into equal groups and count how many in each group	Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups	Use the symbols plus, -, multiplied by, divided by and equals to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. square divided by 2 equals 6, 30 – square equals 24)
Strand 5 – Understanding Shape		
Use familiar objects and common shapes to create and recreate patterns and build models **NRICH:** Chairs and Tables **NRICH:** Repeating Patterns	Visualise and name common 2-D shapes and 3-D solids and describe their features; use them to make patterns, pictures and models **NRICH:** Building with Solid Shapes	Visualise common 2-D shapes and 3-D solids; identify shapes from pictures of them in different positions and orientations; sort, make and describe shapes, referring to their properties **NRICH:** Matching Triangles **NRICH:** Complete the Square **NRICH:** Shadow Play **NRICH:** Skeleton Shapes
Use language such as 'circle' or 'bigger' to describe the shape and size of solids and flat shapes	Identify objects that turn about a point (e.g. scissors) or about a line (e.g. a door); recognise and make whole, half and quarter turns **NRICH:** Turning	Identify reflective symmetry in patterns and 2-D shapes and draw lines of symmetry in shapes
Use everyday words to describe position **NRICH:** Coloured Squares	Visualise and use everyday language to describe the position of objects and direction and distance when moving them, for example when placing or moving objects on a game board **NRICH:** 2 Rings	Follow and give instructions involving position, direction and movement

			Recognise and use whole, half and quarter turns, both clockwise and anticlockwise; know that a right angle represents a quarter turn **NRICH:** Turning Man
Strand 6 - Measuring			
	Use language such as 'greater', 'smaller', 'heavier' or 'lighter' to compare quantities	Estimate, measure, weigh and compare objects, choosing and using suitable uniform non-standard or standard units and measuring instruments (e.g. a lever balance, metre stick or measuring jug) **NRICH:** Sizing Them Up **NRICH:** Wallpaper	Estimate, compare and measure lengths, weights and capacities, choosing and using standard units (m, cm, kg, litre) and suitable measuring instruments **NRICH:** Little Man
	Use everyday language related to time; order and sequence familiar events and measure short periods of time **NRICH:** Snap	Use vocabulary related to time; order days of the week and months; read the time to the hour and half hour	Read the numbered divisions on a scale, and interpret the divisions between them (e.g. on a scale from 0 to 25 with intervals of 1 shown but only the divisions 0, 5, 10, 15 and 20 numbered); use a ruler to draw and measure lines to the nearest centimetre
			Use units of time (seconds, minutes, hours, days) and know the relationships between them; read the time to the quarter hour; identify time intervals, including those that cross the hour **NRICH:** Stop the Clock
Strand 7 - Handling Data			
	Sort familiar objects to identify their similarities and differences	Answer a question by recording information in lists and tables; present outcomes using practical resources, pictures, block graphs or pictograms **NRICH:** Noah	Answer a question by collecting and recording data in lists and tables; represent the data as block graphs or pictograms to show results; use ICT to organise and present data **NRICH:** Ladybird Count
	Count how many objects share a particular property, presenting results using pictures, drawings or numerals	Use diagrams to sort objects into groups according to a given criterion; suggest a different criterion for grouping the same objects **NRICH:** Sort the Street	Use lists, tables and diagrams to sort objects; explain choices using appropriate language, including 'not' **NRICH:** Carroll Diagrams

Appendix 20	for Activity 3	a copy for each teacher (or use it on the website) (7 pages)

NRICH www.nrich.maths.org problems linked to the Framework for teaching mathematics in Years 3, 4, 5 and 6
(N.B. This is work in progress– we would really appreciate your comments. Please email emp1001@cam.ac.uk)

	Year 3	Year 4	Year 5	Year 6	Year 6-7
Strand 1 - Using and Applying	Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations **NRICH: A Square of Numbers**	Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate **NRICH: The Puzzling Sweet Shop**	Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use **NRICH: Money Bags** **NRICH: Amy's Dominoes**	Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use **NRICH: Two Primes Make One Square** **NRICH: What's it Worth?**	Solve problems by breaking down complex calculations into simpler steps; choose and use operations and calculation strategies appropriate to the numbers and context; try alternative approaches to overcome difficulties; present, interpret and compare solutions
	Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem **NRICH: Buying a Balloon**	Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem **NRICH: Sealed Solution** **NRICH: Prison Cells**	Tabulate systematically the information in a problem or puzzle; identify and record the steps or calculations needed to solve it, using symbols where appropriate; interpret solutions in the original context and check their accuracy **NRICH: Counting Cards**	Represent information or unknown numbers in a problem, for example in a table, formula or equation; explain solutions in the context of the problem
	Follow a line of enquiry by deciding what information is important; make and use lists, tables and graphs to organise and interpret the information **NRICH: Sweets in a Box**	Suggest a line of enquiry and the strategy needed to follow it; collect, organise and interpret selected information to find answers	Plan and pursue an enquiry; present evidence by collecting, organising and interpreting information; suggest extensions to the enquiry	Suggest, plan and develop lines of enquiry; collect, organise and represent information, interpret results and review methods; identify and answer related questions	Develop and evaluate lines of enquiry; identify, collect, organise and analyse relevant information; decide how best to represent conclusions and what further questions to ask
	Describe patterns and relationships involving numbers or shapes, and use these to solve problems	Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples	Explore patterns, properties and relationships and propose a general statement involving numbers or shapes; identify examples for which the statement is true or false **NRICH: Up and Down Staircases**	Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of c pens at 15 pence each is 15c pence) **NRICH: Sticky Triangles**	Generate sequences and describe the general term; use letters and symbols to represent unknown numbers or variables; represent simple relationships as graphs

	Describe and explain methods, choices and solutions to puzzles and problems, orally and in writing, using pictures and diagrams	Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols	Explain reasoning using diagrams, graphs and text; refine ways of recording using images and symbols	Explain reasoning and conclusions, using words, symbols or diagrams as appropriate **NRICH: Make 37** **NRICH: Got It!**	Explain and justify reasoning and conclusions, using notation, symbols and diagrams; find a counter-example to disprove a conjecture; use step-by-step deductions to solve problems involving shapes
Strand 2 - Counting and Understanding Number	Read, write and order whole numbers to at least 1000 and position them on a number line; count on from and back to zero in single-digit steps or multiples of 10	Recognise and continue number sequences formed by counting on or back in steps of constant size	Count from any given number in whole-number and decimal steps, extending beyond zero when counting backwards; relate the numbers to their position on a number line **NRICH: Swimming Pool** **NRICH: Tug Harder!** **NRICH: First Connect Three**	Find the difference between a positive and a negative integer, or two negative integers, in context **NRICH: Consecutive Numbers** **NRICH: Sea Level**	Compare and order integers and decimals in different contexts
	Partition three-digit numbers into multiples of 100, 10 and 1 in different ways	Partition, round and order four-digit whole numbers; use positive and negative numbers in context and position them on a number line; state inequalities using the symbols less than and greater than (e.g. -3 greater than -5, -1 less than plus1)	Explain what each digit represents in whole numbers and decimals with up to two places, and partition, round and order these numbers	Use decimal notation for tenths, hundredths and thousandths; partition, round and order decimals with up to three places, and position them on the number line	Order a set of fractions by converting them to decimals
	Round two-digit or three-digit numbers to the nearest 10 or 100 and give estimates for their sums and differences	Use decimal notation for tenths and hundredths and partition decimals; relate the notation to money and measurement; position one-place and two-place decimals on a number line	Express a smaller whole number as a fraction of a larger one (e.g. recognise that 5 out of 8 is five eighths); find equivalent fractions (e.g. seven tenths equals fourteen twentieths, or nineteen tenths equals 1nine tenths); relate fractions to their decimal representations	Express a larger whole number as a fraction of a smaller one (e.g. recognise that 8 slices of a 5-slice pizza represents eight fifths or 1three fifths pizzas); simplify fractions by cancelling common factors; order a set of fractions by converting them to fractions with a common denominator **NRICH: Chocolate**	Recognise approximate proportions of a whole and use fractions and percentages to describe and compare them, for example when interpreting pie charts
	Read and write proper fractions (e.g. three sevenths, nine tenths), interpreting the denominator as the parts of a whole and the numerator as the number of parts; identify and estimate fractions of shapes; use diagrams to compare fractions and establish equivalents	Recognise the equivalence between decimal and fraction forms of one half, quarters, tenths and hundredths	Understand percentage as the number of parts in every 100 and express tenths and hundredths as percentages	Express one quantity as a percentage of another (e.g. express pound400 as a percentage of pound1000); find equivalent percentages, decimals and fractions	Use ratio notation, reduce a ratio to its simplest form and divide a quantity into two parts in a given ratio; solve simple problems involving ratio and direct proportion (e.g. identify the quantities needed to make a fruit drink by mixing water and juice in a given ratio)

		Use diagrams to identify equivalent fractions (e.g. six eighths and three quarters, or seventy hundredths and seven tenths); interpret mixed numbers and position them on a number line (e.g. 3 one half)	Use sequences to scale numbers up or down; solve problems involving proportions of quantities (e.g. decrease quantities in a recipe designed to feed six people) **NRICH:** Blackcurrantiest	Solve simple problems involving direct proportion by scaling quantities up or down **NRICH:** Orange Drink **NRICH:** Pumpkin Pie Problem	
		Use the vocabulary of ratio and proportion to describe the relationship between two quantities (e.g. 'There are 2 red beads to every 3 blue beads, or 2 beads in every 5 beads are red'); estimate a proportion (e.g. 'About one quarter of the apples in the box are green')			
Strand 3 - Knowing and Using Number Facts	Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100	Use knowledge of addition and subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000	Use knowledge of place value and addition and subtraction of two-digit numbers to derive sums and differences and doubles and halves of decimals (e.g. 6.5 plus over minus 2.7, half of 5.6, double 0.34)	Use knowledge of place value and multiplication facts to 10 multiplied by 10 to derive related multiplication and division facts involving decimals (e.g. 0.8 multiplied by 7, 4.8 divided by 6)	Consolidate rapid recall of number facts, including multiplication facts to 10 multiplied by 10 and the associated division facts
	Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000 **NRICH:** Growing Garlic	Identify the doubles of two-digit numbers; use these to calculate doubles of multiples of 10 and 100 and derive the corresponding halves	Recall quickly multiplication facts up to 10 multiplied by 10 and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts	Use knowledge of multiplication facts to derive quickly squares of numbers to 12multiplied by12 and the corresponding squares of multiples of 10 **NRICH:** One Wasn't Square	Recognise the square roots of perfect squares to 12 multiplied by 12
	Use knowledge of number operations and corresponding inverses, including doubling and halving, to estimate and check calculations	Derive and recall multiplication facts up to 10 multiplied by 10, the corresponding division facts and multiples of numbers to 10 up to the tenth multiple **NRICH:** Multiplication Square Jigsaw **NRICH:** Shape Times Shape **NRICH:** What do you Need?	Identify pairs of factors of two-digit whole numbers and find common multiples (e.g. for 6 and 9) **NRICH:** Multiples Grid **NRICH:** Music to my Ears **NRICH:** Multiplication Squares **NRICH:** Flashing Lights	Recognise that prime numbers have only two factors and identify prime numbers less than 100; find the prime factors of two-digit numbers **NRICH:** Factors and Multiples Game	Recognise and use multiples, factors, divisors, common factors, highest common factors and lowest common multiples in simple cases **NRICH:** What's in the Box? **NRICH:** Factor-Multiple Chains **NRICH:** The Moons of Vuvv
		Use knowledge of rounding, number operations and inverses to estimate and check calculations	Use knowledge of rounding, place value, number facts and inverse operations to estimate and check calculations	Use approximations, inverse operations and tests of divisibility to estimate and check results	Make and justify estimates and approximations to calculations
		Identify pairs of fractions that total 1			

Strand 4 - Calculating	Add or subtract mentally combinations of one-digit and two-digit numbers **NRICH:** Super Shapes	Add or subtract mentally pairs of two-digit whole numbers (e.g. 47 plus 58, 91 - 35) **NRICH:** Twenty Divided Into Six	Extend mental-methods for whole-number calculations, for example to multiply a two-digit by a one-digit number (e.g. 12 multiplied by 9), to multiply by 25 (e.g. 16 multiplied by 25), to subtract one near-multiple of 1000 from another (e.g. 6070 - 4097)	Calculate mentally with integers and decimals: U.t plus over minus U.t, TU multiplied by U, TU divided by U, U.t multiplied by U, U.t divided by U	Understand how the commutative, associative and distributive laws, and the relationships between operations, including inverse operations, can be used to calculate more efficiently; use the order of operations, including brackets
	Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers	Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and pound.p	Use efficient written methods to add and subtract whole numbers and decimals with up to two places **NRICH:** Reach 100	Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer	Consolidate and extend mental methods of calculation to include decimals, fractions and percentages **NRICH:** Route Product
	Multiply one-digit and two-digit numbers by 10 or 100, and describe the effect	Multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), understanding the effect; relate to scaling up or down **NRICH:** The Deca Tree	Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000	Relate fractions to multiplication and division (e.g. 6 divided by 2 equals one half of 6 equals 6 multiplied by one half); express a quotient as a fraction or decimal (e.g. 67 divided by 5 equals 13.4 or 13two fifths); find fractions and percentages of whole-number quantities **NRICH:** Andy's Marbles **NRICH:** Would you Rather? **NRICH:** Forgot the Numbers	Use standard column procedures to add and subtract integers and decimals, and to multiply two-digit and three-digit integers by a one-digit or two-digit integer; extend division to dividing three-digit integers by a two-digit integer **NRICH:** Two and Two **NRICH:** Trebling **NRICH:** All the Digits
	Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 multiplied by 3, 50 divided by 4); round remainders up or down, depending on the context	Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 multiplied by 9, 98 divided by 6)	Refine and use efficient written methods to multiply and divide HTU multiplied by U, TU multiplied by TU, U.t multiplied by U and HTU divided by U	Use a calculator to solve problems involving multi-step calculations	Calculate percentage increases or decreases and fractions of quantities and measurements (integer answers)
	Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences **NRICH:** Secret Number	Find fractions of numbers, quantities or shapes (e.g. one fifth of 30 plums, three eighths of a 6 by 4 rectangle) **NRICH:** A Bowl of Fruit **NRICH:** Fractional Triangles	Find fractions using division (e.g. one hundredth of 5 kg), and percentages of numbers and quantities (e.g. 10percent, 5percent and 15percent of pound80)		Use bracket keys and the memory of a calculator to carry out calculations with more than one step; use the square root key

	Find unit fractions of numbers and quantities (e.g. one half, one third, one quarter and one sixth of 12 litres) **NRICH:** Fair Feast	Use a calculator to carry out one-step and two-step calculations involving all four operations; recognise negative numbers in the display, correct mistaken entries and interpret the display correctly in the context of money	Use a calculator to solve problems, including those involving decimals or fractions (e.g. find three quarters of 150 g); interpret the display correctly in the context of measurement		
Strand 5 - Understanding Shape	Relate 2-D shapes and 3-D solids to drawings of them; describe, visualise, classify, draw and make the shapes **NRICH:** Building Blocks **NRICH:** The Third Dimension	Draw polygons and classify them by identifying their properties, including their line symmetry **NRICH:** Let's Reflect	Identify, visualise and describe properties of rectangles, triangles, regular polygons and 3-D solids; use knowledge of properties to draw 2-D shapes, and to identify and draw nets of 3-D shapes **NRICH:** Square It **NRICH:** Cut Nets	Describe, identify and visualise parallel and perpendicular edges or faces; use these properties to classify 2-D shapes and 3-D solids **NRICH:** Where Are They?	Use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes
	Draw and complete shapes with reflective symmetry; draw the reflection of a shape in a mirror line along one side	Visualise 3-D objects from 2-D drawings; make nets of common solids **NRICH:** A Puzzling Cube	Read and plot coordinates in the first quadrant; recognise parallel and perpendicular lines in grids and shapes; use a set-square and ruler to draw shapes with perpendicular or parallel sides	Make and draw shapes with increasing accuracy and apply knowledge of their properties **NRICH:** Stringy Quads **NRICH:** Making Cuboids	Extend knowledge of properties of triangles and quadrilaterals and use these to visualise and solve problems, explaining reasoning with diagrams **NRICH:** Nine-pin Triangles **NRICH:** Transformations on a Pegboard **NRICH:** Cut it Out **NRICH:** Quadrilaterals
	Read and record the vocabulary of position, direction and movement, using the four compass directions to describe movement about a grid	Recognise horizontal and vertical lines; use the eight compass points to describe direction; describe and identify the position of a square on a grid of squares **NRICH:** Square Corners	Complete patterns with up to two lines of symmetry; draw the position of a shape after a reflection or translation	Visualise and draw on grids of different types where a shape will be after reflection, after translations, or after rotation through 90degrees or 180degrees about its centre or one of its vertices	Know the sum of angles on a straight line, in a triangle and at a point, and recognise vertically opposite angles
	Use a set-square to draw right angles and to identify right angles in 2-D shapes; compare angles with a right angle; recognise that a straight line is equivalent to two right angles	Know that angles are measured in degrees and that one whole turn is 360degrees; compare and order angles less than 180degrees	Estimate, draw and measure acute and obtuse angles using an angle measurer or protractor to a suitable degree of accuracy; calculate angles in a straight line **NRICH:** Six Places to Visit	Use coordinates in the first quadrant to draw, locate and complete shapes that meet given properties **NRICH:** A Cartesian Puzzle **NRICH:** Eight Hidden Squares	Use all four quadrants to find coordinates of points determined by geometric information **NRICH:** Coordinate Tan **NRICH:** Ten Hidden Squares

				Estimate angles, and use a protractor to measure and draw them, on their own and in shapes; calculate angles in a triangle or around a point **NRICH:** How Safe Are You?	Identify all the symmetries of 2-D shapes; transform images using ICT **NRICH:** Symmetry Challenge **NRICH:** Coordinate Challenge
					Construct a triangle given two sides and the included angle
Strand 6 - Measuring	Know the relationships between kilometres and metres, metres and centimetres, kilograms and grams, litres and millilitres; choose and use appropriate units to estimate, measure and record measurements	Choose and use standard metric units and their abbreviations when estimating, measuring and recording length, weight and capacity; know the meaning of 'kilo', 'centi' and 'milli' and, where appropriate, use decimal notation to record measurements (e.g. 1.3 m or 0.6 kg)	Read, choose, use and record standard metric units to estimate and measure length, weight and capacity to a suitable degree of accuracy (e.g. the nearest centimetre); convert larger to smaller units using decimals to one place (e.g. change 2.6 kg to 2600 g)	Select and use standard metric units of measure and convert between units using decimals to two places (e.g. change 2.75 litres to 2750 ml, or vice versa)	Convert between related metric units using decimals to three places (e.g. convert 1375 mm to 1.375 m, or vice versa)
	Read, to the nearest division and half-division, scales that are numbered or partially numbered; use the information to measure and draw to a suitable degree of accuracy	Interpret intervals and divisions on partially numbered scales and record readings accurately, where appropriate to the nearest tenth of a unit	Interpret a reading that lies between two unnumbered divisions on a scale	Read and interpret scales on a range of measuring instruments, recognising that the measurement made is approximate and recording results to a required degree of accuracy; compare readings on different scales, for example when using different instruments	Solve problems by measuring, estimating and calculating; measure and calculate using imperial units still in everyday use; know their approximate metric values
	Read the time on a 12-hour digital clock and to the nearest 5 minutes on an analogue clock; calculate time intervals and find start or end times for a given time interval **NRICH:** Two Clocks	Draw rectangles and measure and calculate their perimeters; find the area of rectilinear shapes drawn on a square grid by counting squares **NRICH:** Torn Shapes	Draw and measure lines to the nearest millimetre; measure and calculate the perimeter of regular and irregular polygons; use the formula for the area of a rectangle to calculate the rectangle's area **NRICH:** Fitted	Calculate the perimeter and area of rectilinear shapes; estimate the area of an irregular shape by counting squares **NRICH:** Numerically Equal	Calculate the area of right-angled triangles given the lengths of the two perpendicular sides, and the volume and surface area of cubes and cuboids **NRICH:** Brush Loads **NRICH:** More Transformations on a Pegboard

		Read time to the nearest minute; use am, pm and 12-hour clock notation; choose units of time to measure time intervals; calculate time intervals from clocks and timetables **NRICH: Wonky Watches** **NRICH: Clocks**	Read timetables and time using 24-hour clock notation; use a calendar to calculate time intervals **NRICH: How Many Times?** **NRICH: 5 on the Clock**		
Strand 7 - Handling Data	Answer a question by collecting, organising and interpreting data; use tally charts, frequency tables, pictograms and bar charts to represent results and illustrate observations; use ICT to create a simple bar chart	Answer a question by identifying what data to collect; organise, present, analyse and interpret the data in tables, diagrams, tally charts, pictograms and bar charts, using ICT where appropriate	Describe the occurrence of familiar events using the language of chance or likelihood	Describe and predict outcomes from data using the language of chance or likelihood **NRICH: Domino Pick** **NRICH: Odds or Sixes?** **NRICH: Twelve Pointed Star Game** **NRICH: Same or Different?**	Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts **NRICH: Roll These Dice**
	Use Venn diagrams or Carroll diagrams to sort data and objects using more than one criterion **NRICH: Venn Diagrams** **NRICH: More Carroll Diagrams**	Compare the impact of representations where scales have intervals of differing step size	Answer a set of related questions by collecting, selecting and organising relevant data; draw conclusions, using ICT to present features, and identify further questions to ask **NRICH: Real Statistics**	Solve problems by collecting, selecting, processing, presenting and interpreting data, using ICT where appropriate; draw conclusions and identify further questions to ask **NRICH: It's a Tie**	Explore hypotheses by planning surveys or experiments to collect small sets of discrete or continuous data; select, process, present and interpret the data, using ICT where appropriate; identify ways to extend the survey or experiment
			Construct frequency tables, pictograms and bar and line graphs to represent the frequencies of events and changes over time	Construct and interpret frequency tables, bar charts with grouped discrete data, and line graphs; interpret pie charts **NRICH: Match the Matches**	Construct, interpret and compare graphs and diagrams that represent data, for example compare proportions in two pie charts that represent different totals
			Find and interpret the mode of a set of data	Describe and interpret results and solutions to problems using the mode, range, median and mean	Write a short report of a statistical enquiry and illustrate with appropriate diagram