Investigation of the double-angle formulae

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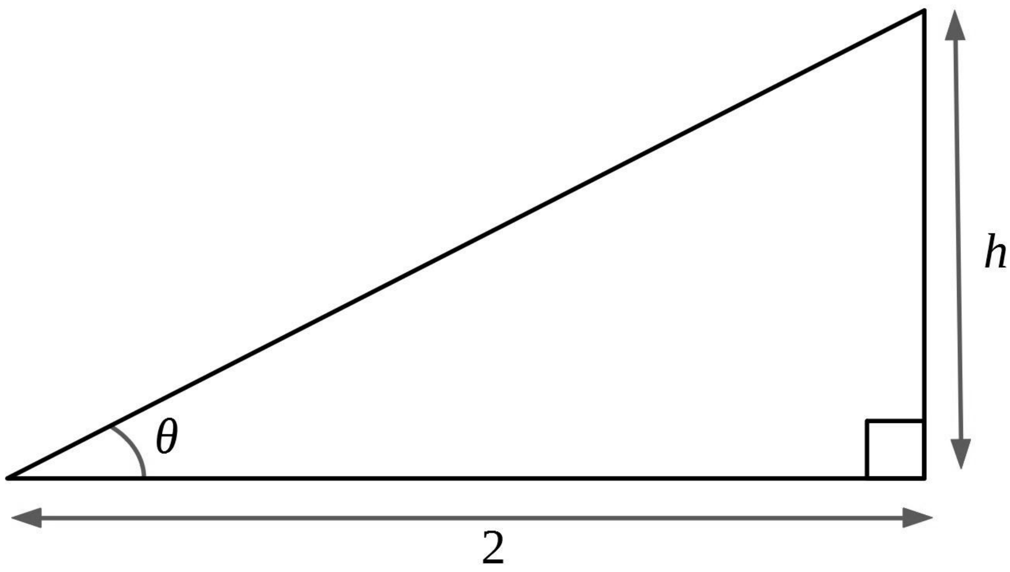
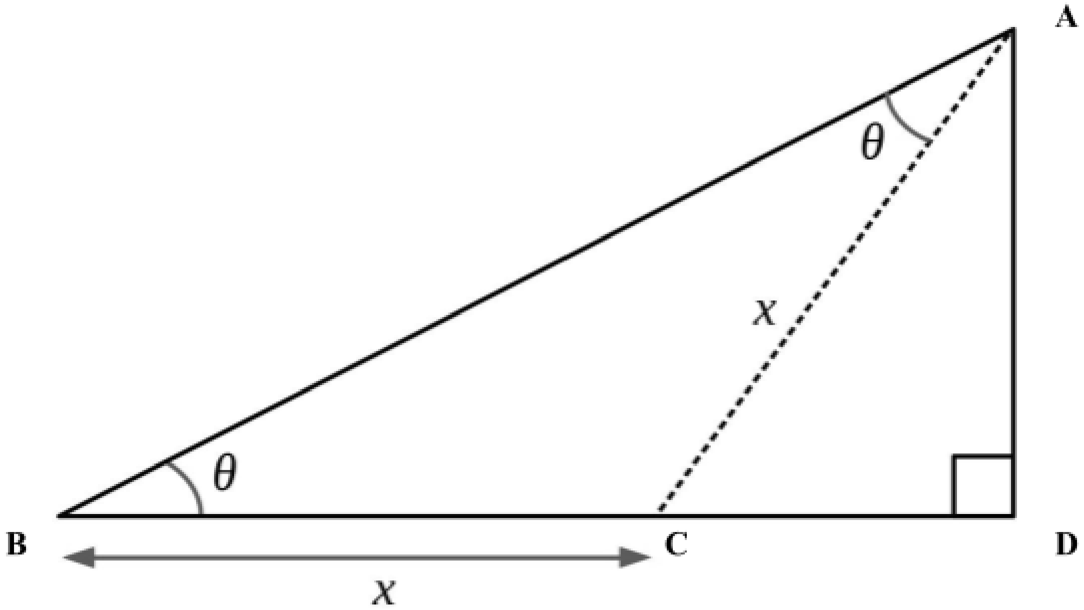
Abstract: This paper explores the proof of the double-angle formulae.

1 Height of the triangle



The height of the triangle is equal to $2tan\theta = 2t$.

2 Proof of the double-angle formulae



As the sum of interior angles of a triangle is equal to 180°, the angle ACB is equal to $180 - 2\theta$. As the sum of angles on a straight line is equal to 180°, the angle ACD is equal to 2θ. As the length BD is equal to 2, the length CD is equal to $2 - x$. The length AD is equal to $2t$.

The trigonometric ratios with respect to the angle ACD,

$tan2\theta = \frac{2t}{2-x}$

$sin2\theta = \frac{2t}{x}$

$cos2\theta = \frac{2-x}{x}$

The Pythagorean theorem with respect to the lengths AC, CD and AD,

$x^2 = (2 - x)^2 + (2t)^2$

$x^2 = x^2 - 4x + 4 + 4t^2$

$4x = 4 + 4t^2$

Using the substitution $x = 1 + t^2$ to prove the double angle formulae,

$tan2\theta = \frac{2t}{2-(1+t^2)} = \frac{2t}{1-t^2}$

$sin2\theta = \frac{2t}{1+t^2}$

$cos2\theta = \frac{2-(1+t^2)}{1+t^2} = \frac{1-t^2}{1+t^2}$