Solution for Diagonal in a Spiral

Myself, Shubhangee, had worked in Ganit Kreeda, Vicharvatika, India with 15 kids, Viha, Abhiram, Eshann, Anirved, Arya, Rivaan, Miraya, Asma, Aprameya, Vibha, Rudraraj, Nithyashree, Adhrit, Kathir, Arnav, Arjun and Harshad worked on **Diagonal in a Spiral.**

CHALLENGE 1

Find the numbers that would be in this green upper-left to lower-right diagonal for a spiral going up to 144 instead of just 16.

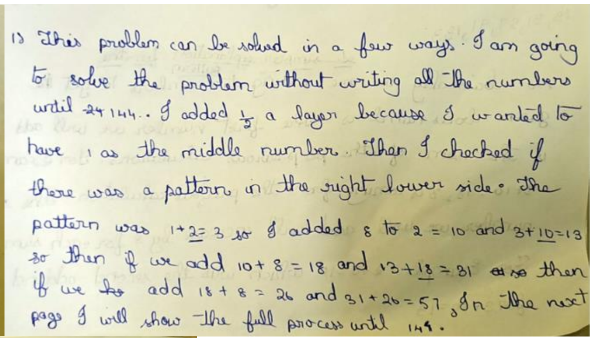
Attaching the explanation given by Asma.

Asma P Page no: 1 Ganit Kreeda

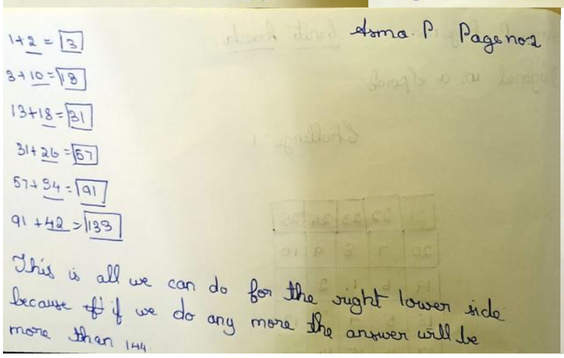
Diagonal in a Spiral:

Challenge : 1

21	22	23	24	25
20	7	8	9	10
19	6	1	2	11
18	5	4	3	12
17	16	15	14	13



1) This problem can be solved in a few ways. I am going to solve the problem without writing all the numbers until ~~24~~ 144.. I added ½ a layer because I wanted to have 1 as the middle number. Then I checked if there was a pattern in the right lower side. The pattern was 1+2=3 so I added 8 to 2 = 10 and 3+10=13 so then if we add 10+8=18 and 13+18=31 ~~so~~ then if we ~~to~~ add 18+8=26 and 31+26=57, In the next page I will show the full process until 144.



Asma P Page no 2

1+2 = $\boxed{3}$

3+10 = $\boxed{13}$

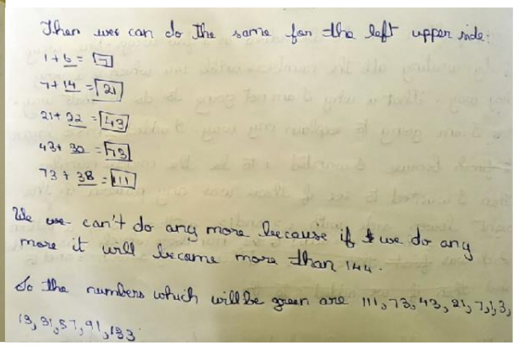
13+18 = $\boxed{31}$

31+26 = $\boxed{57}$

57+34 = $\boxed{91}$

91+42 = $\boxed{133}$

This is all we can do for the right lower side because ~~if~~ if we do any more the answer will be more than 144



Then we can do the same for the left upper side:

1+6 = $\boxed{7}$

7+14 = $\boxed{21}$

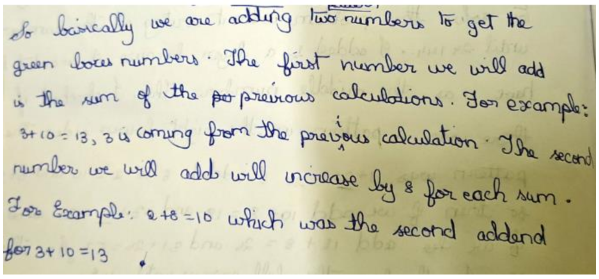
21+22 = $\boxed{43}$

43+30 = $\boxed{73}$

73+38 = $\boxed{111}$

We ~~we~~ can't do any more because if ~~t~~ we do any more it will become more than 144.

So the numbers which will be green are 111, 73, 43, 21, 7, 1, 3, 13, 31, 57, 91, 133.



So basically we are adding two numbers to get the green box numbers. The first number we will add is the sum of the ~~pr~~ previous calculations. For example: 3+10=13, 3 is coming from the previous calculation. The second number we will add will increase by 8 for each sum. For Example: 2+8=10 which was the second addend for 3+10=13

Similar explanation was given by Anirved.

He found out that 1+2=3, 3+ (2x2) =7, 7+(2x3) = 3, 13+(2x4) =21 and so on…using this logic Viha found out all the numbers in green diagonal (less than 144) without listing down any other numbers.

Aprameya found out all the numbers in a Spiral and then observed different patterns.

- 11^2 121-10
- 9^2 81-8
- 7^2 49-6
- 5^2 25-4
- 3^2 9-2
- 2^2 4-1
- 4^2 16-3
- 6^2 36-5
- 8^2 64-7
- 10^2 100-9
- 12^2 144-11

111 11^2-10	112	113	114	115	116	117	118	119	120	121	122
110	73 $=9^2-8$	74	75	76	77	78	79	80	81	82	123
109	72	43 $=7^2-6$	44	45	46	47	48	49	50	83	124
108	71	42	21 $=5^2-4$	22	23	24	25	26	51	84	125
107	70	41	20	7 $=3^2-2$	8	9	10	27	52	85	126
106	69	40	19	6	1	2	11	28	53	86	127
105	68	39	18	5	4 2^2-1	3	12	29	54	87	128
104	67	38	17	16 4^2	15	14 $4^2-3=$	13	30	55	88	129
103	66	37	36 6^2	35	34	33	32 6^2-5	31	56	89	130
102	65	64 8^2	63	62	61	60	59	58 8^2-7	57	90	131
101	100 10^2	99	98	97	96	95	94	93	92 $10^2-9=$	91	132
144 12^2	143	142	141	140	139	138	137	136	135	134 12^2-11	133

Kids spotted many different patterns by observing the grid.
Kids spotted that the numbers in the lower diagonal are related to sq of even numbers as shown.
Every time they subtracted odd numbers in increasing order.

2^2 – 1 = 4-1 = 3
4^2 – 1 = 16-3 = 13
6^2 – 1 = 36-5 = 31
8^2 – 1 = 64-7 = 57
10^2 – 1 = 100-9 = 91
12^2 – 1 = 144-11= 133

Kids also spotted that the numbers in the upper diagonal are related to sq of odd numbers as shown.
Every time they subtracted even numbers in increasing order.

3^2 – 1 = 9-2 = 7
5^2 – 1 = 25-4 = 21
7^2 – 1 = 49-6 = 43
9^2 – 1 = 81-8 = 73
11^2 – 1 = 121-10 =111

Challenge 2

The totals we got for each three are:

227, 137, 71, 29, 11, 17, 47, 101, 179, 281

Challenge 3a

You now need to use the numbers you got from adding the diagonal up in threes.
Use these numbers to make a total that has a 2 as the ones digit. You can only use a number once in any addition.
Do this in as many different ways as possible.

Kids shared different approaches about how they covered all the possibilities.

i) Asma thought in a systematic way with just 2 numbers that gives the sum ending in 2. Then she listed down all the possibilities with 3 no.s, 4 no.s, 5 no.s & so on.
ii) Anirved, Eshaan and Adhrit systematically listed down all the ways they can get for sum ending in 2.
iii) Harshad and Asma gave 5 logical steps to get sum ending in 2:
    - A number which ends with 1 + another number which ends with 1.
    - 3 X A number which ends with 7 + a number which ends with 1.
    - 2 X A number which ends with 9 + 2 X a number which ends with 7.
    - 3 X A number which ends with 7 + 2 X a number which ends with 1 + a number which ends with 9.
    - 3 X A number which ends with 1 + a number that ends with 9.
    - 4 X A number which ends with 7 + 4 x a number which ends with 1.
    - 2 x A number which ends with 9 + 4 x a number which ends with 1.

We used **counting techniques** to calculate for each one as:

- As we have 4 numbers ending in 1 and we need to choose any 2 from this. We can do it in 3+2+1=6 ways.
- As we have 4 numbers ending in 7 and choosing any 3 from this can be done in 4 ways. And a number ending in 1 can be chosen in 4 ways. So, total number of ways = 4x4=16.
- 2 numbers ending in 9 can be chosen in 1 way and 2 numbers ending in 7 can be chosen in 6 ways. So, total number of ways = 1 x 6 = 6 ways.
- As explained earlier this can be done in 4 x 6 x 2 = 48 ways.
- As we have 4 numbers ending in 1 and choosing any 3 from **this is same as leaving one number** and it can be done in 4 ways and a number ending in 9 can be chosen in 2 ways. So, total no. of ways = 4x2=8 ways.
- This can be done in 1 way.
- This can also be done in only 1 way.
  Total number of ways to get sum ending in 2 = 6+16+6+48+8+1+1 = 75 ways.

Adhrit shared one more way to see if all the answers are covered as:

Try to get 2 / 12 / 22/ 32/ 42.. as sum using units place digit. This was very powerful technique and we used this for cross checking the answers.

Finally, we got 75 solutions for challenge 3(a).

The task was very thoughtfully completed for challenge 3(a).

Challenge 3b

Do the same as in Challenge 3a but now the ones digit has to be an 8.
How many different ways are possible?
Here are the points kids have used for 3(b):

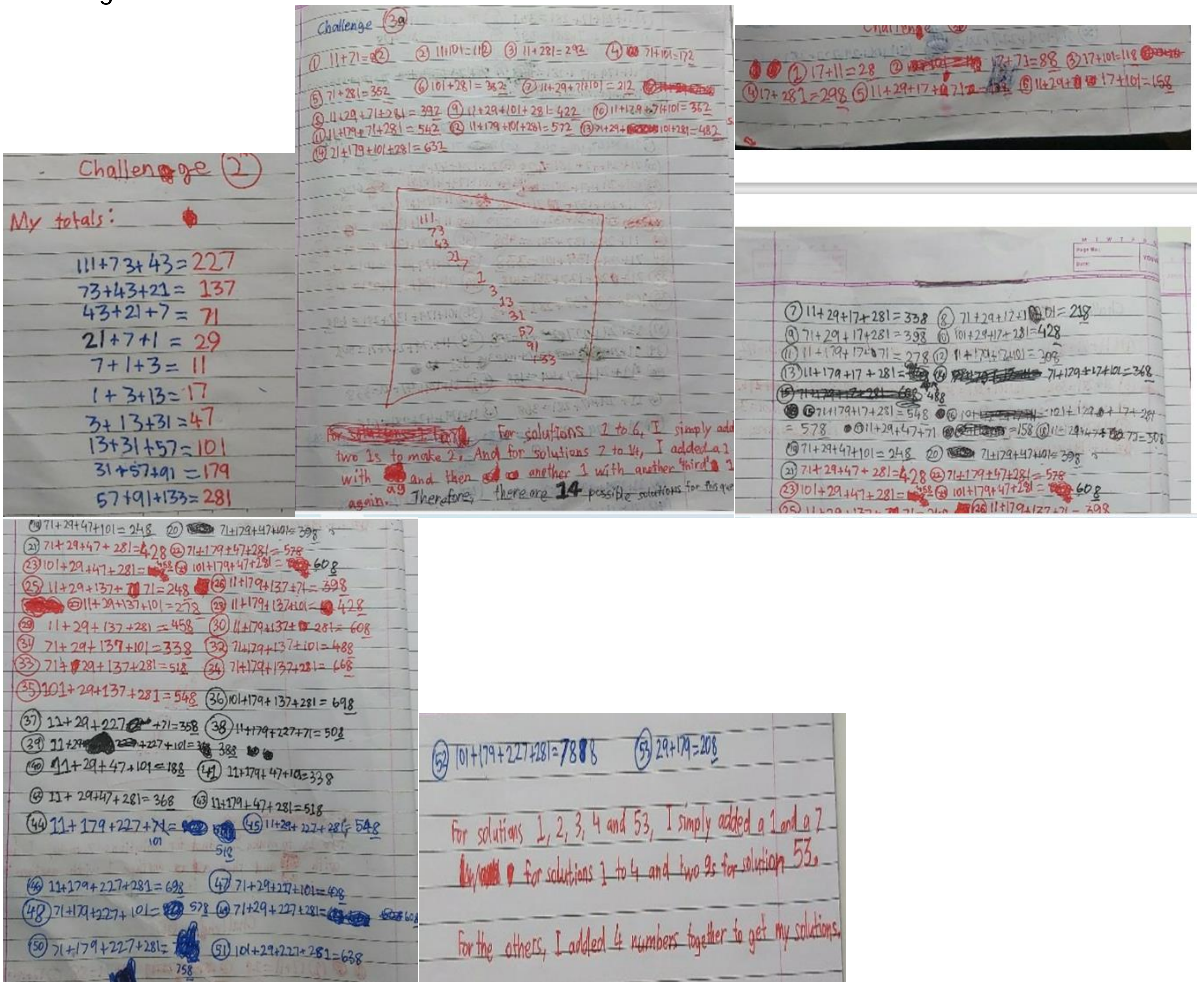
There are 5 ways to do this:

- **A number ending in 7 + a number ending in 1.**
  4x4=16 ways.
- **A number ending in 9 + a number ending in 7 + 2 X (a number ending in 1).**
  2 x 4 x 6 = 48 ways.
- **2 x (A number ending in 9).**
  1 way
- **2 x (A number ending in 7) + 4 X (a number ending in 1).**
  6 x 1 = 6 ways.
- **2 x (A number ending in 9) + 3 X (a number which ending in 1) + a number ending in 7.**
  1 x 4 x 4 = 16 ways.
- **4 x(A number ending in 7)**
  1 way
- **4 x (A number ending in 7) + A number ending in 9 + a number ending in 1.**
  1 x 2 x 4 = 8 ways.
- **4 x (A number ending in 7) + 2x (A number ending in 9) + 2x (a number ending in 1).**
  1 x 1 x 6 = 6 ways

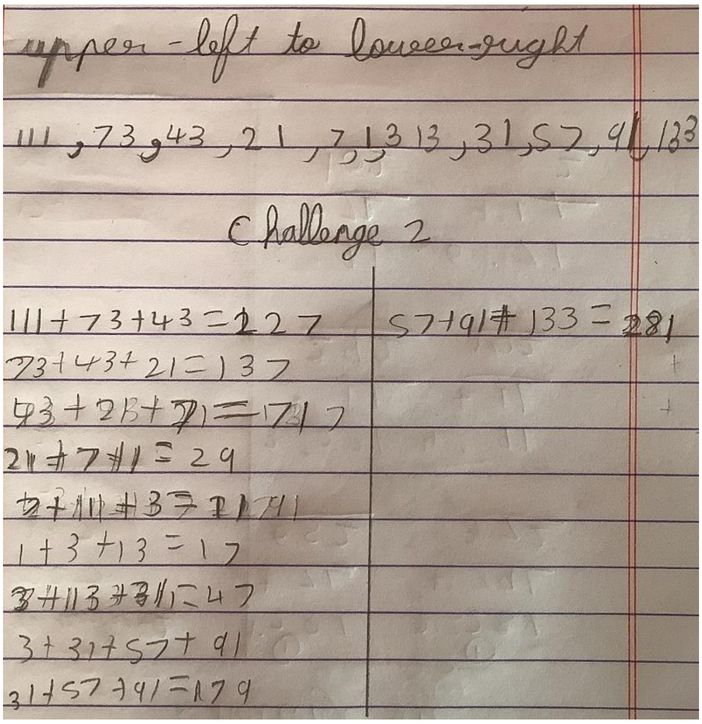
Total number of ways to get sum ending in 8 = 16+48+1+6+16+1+8+6= 102 ways.

Here also we used the similar technique for cross checking as try to get 8 / 18 / 28/ 38/ 48.. as sum using units place digit.

Attaching Anirved work as it is:



Attaching Adhrit's work as it is:



Challenge 3A

Numbers
227
137
71
29
11
17
47
91
174
281

1. 281 + 71 = 652
2. 281 + 91 = 372
3. 281 + 11 = 292
4. 91 + 11 = 102
5. 91 + 71 = 162
6. 71 + 11 = 82

7. 227 + 137 + 71 + 17 = 452
8. 227 + 137 + 17 + 11 = 392
9. 227 + 137 + 17 + 91 = 472
10. 227 + 137 + 17 + 281 = 662
11. 227 + 137 + 47 + 71 = 482
12. 227 + 137 + 47 + 11 = 422

13. 227 + 137 + 47 + 91 = 502
14. 227 + 137 + 47 + 281 = 692
15. 227 + 47 + 17 + 71 = 362
16. 227 + 47 + 17 + 11 = 302

17. 227 + 47 + 17 + 91 = 382
18. 227 + 47 + 17 + 281 = 572
19. 137 + 47 + 17 + 71 = 272

Challenge

20. 137 + 47 + 17 + 11 = 212
21. 137 + 47 + 17 + 91 = 292
22. 137 + 47 + 17 + 281 = 482

Challenge 3B

Numbers
227
137
71
29
11
17
47
91
179
281

1· 227
71
298

2· 227
11
238

3· 1 227
\+ 91
318

4· 1 227
281
508

1
5· 137
\+ 71
208

6· 137
\+ 11
148

7· 1 137 8· 1 137

7· 1 137
\+ 91
228

8· 1 137
\+ 281
418

9· 17
\+ 71
88

10· 17
\+ 11
28

11· 17
\+ 91
108

12· 17
\+ 281
298

13· 47
\+ 71
118

14· 47
\+ 11
58

15· 47
\+ 91
138

16· 1 47
\+ 281
328