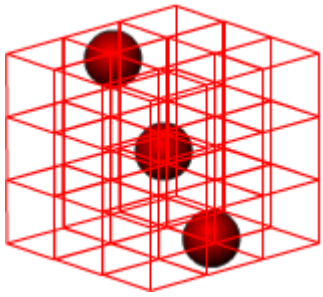
Caroline's method



All winning lines must pass either:

- along an edge of the cube
- through the middle of a face
- through the centre of the cube

There are 12 edges on a cube so there are 12 winning lines **along edges**.

There are 6 faces on a cube, and 4 winning lines that pass through the middle of each face, so there are 24 winning lines **through the middle of faces**.

Finally we need to consider the winning lines that go **through the centre cube**:

vertex to opposite vertex: 4
middle of edge to middle of opposite edge: 6
middle of face to middle of opposite face: 3

In total, there are 12+24+4+6+3=49 winning lines.

Grae's method

The winning lines may be counted by looking at lines:

- in each horizontal plane
- in each vertical plane from left to right
- in each vertical plane from front to back
- in the diagonal planes

On a plane there are 8 winning lines.
In the cube, there are 3 **horizontal planes**, so 8×3=24 winning lines.

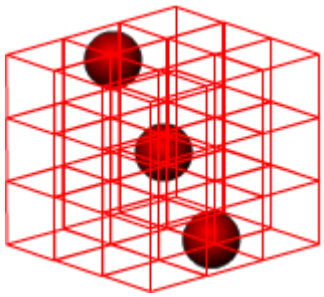
There are also 3 **vertical planes going from left to right**, but now with only 5 **new** winning lines per plane, as the 3 horizontal lines have already been counted. So 5×3=15 winning lines.

On the 3 **vertical planes going from front to back**, we now only have 2 **new** (diagonal) winning lines per plane. So 2×3=6 winning lines.

Finally, there are also **diagonal planes** to consider. There are 4 winning lines going from corner to diagonally opposite corner.

In total, there are 24+15+6+4=49 winning lines.

Alison's method



There are 3 possible places where a line can start:

- at a vertex
- at the middle of an edge
- in the centre of a face

A cube has 8 vertices, 12 edges and 6 faces.

From a **vertex** there are 7 other vertices that you can join to in order to make a winning line. $7\times8=56$ lines, but this counts each line from both ends, so there are 28 'vertex' winning lines.

From the **middle of an edge** there are 3 other middles-of-edges that you can join to in order to make a winning line. $3\times12=36$ lines, but this counts each line from both ends so there are 18 'middle of edge' winning lines.

From the **centre of each face** there is one winning line, joining to the opposite face, so there are 3 'centre of face' winning lines.

So in total, there are $28+18+3=49$ winning lines.

James' method

Winning lines can either be:

- Diagonal
- Not diagonal

Considering the **non-diagonal winning lines** first:
There are 9 from front to back.
There are 9 from left to right.
There are 9 from top to bottom.

Considering the **diagonal winning lines:**

On each layer there are 2 diagonal winning lines so:
There are 6 from front to back.
There are 6 from top to bottom.
There are 6 from left to right.

There are 4 lines from a vertex to a diagonally opposite vertex.

In total, there are $27+18+4=49$ winning lines.